Liquidity, Debt Denomination, and Currency Dominance

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Abstract

The international monetary system of the last four centuries has experienced the rise, persistence, and fall of specific currencies as the dominant unit of denomination in global debt contracts. We provide a liquidity-based theory to explain this pattern. Firms issue debt that can be extinguished by trading their revenues for financial assets of the same denomination. When asset markets differ in their liquidity, as modeled via endogenous search frictions, firms optimally choose to denominate their debt in the unit of the asset that is most liquid. Equilibria with a single dominant currency emerge from a positive feedback cycle whereby issuing in the more liquid denomination endogenously raises the benefits of that denomination. This feedback mechanism has historically been seeded by governments that created the largest pool of liquid assets in the same denomination. Once dominance is established, a country’s costs of investing in the ability to create liquid assets, such as by increasing fiscal capacity, are lower while the incentives to do so are higher, thereby entrenching dominance. We explain the historical experiences of the Dutch florin, the British pound sterling, the US dollar, and the transitions between them. Our theory highlights normative features of liquidity provision in the international monetary system through the lens of the Bretton Woods arrangement, and we discuss the implications of modern policy tools such as central bank swap lines. We rationalize the current dollar-dominant international financial architecture and provide predictions about the potential rise of the Chinese renminbi.

Keywords: International Monetary System, Dollar Dominance, Liquidity, Corporate Debt.

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1 Introduction

The international monetary system has featured few but persistent dominant currencies in the denomination of financial contracts. The first dominant currency, the Dutch florin of the 17th century (Quinn and Roberds 2014b), was followed by the British pound sterling in the 19th century (Lindert 1969; Eichengreen 2005), which in turn was replaced by the US dollar in the 20th century. This paper posits a theory for why, among many alternatives for debt denomination, one currency emerges endogenously as dominant in equilibrium and is able to retain that dominance for prolonged periods. Our theory proposes that debt market liquidity is a key component of the factors establishing currency dominance in the denomination of global debt contracts.

We begin by observing that when borrowers such as firms issue debt, they must choose a unit of denomination for the contract. This choice determines the asset that the borrower must hand over to extinguish the debt at the time of settlement. The issuer, for example a farmer, could issue debt that is denominated in any arbitrary unit, such as bushels of wheat. In this case, upon maturity the farmer would be obliged to deliver bushels of wheat. But if bushels of wheat are difficult to come by—that is, if the wheat market is illiquid—this would be a costly decision. Perhaps a wheat farmer would issue such debt, but a manufacturer would not. Hence the settlement benefits of asset market liquidity, which we model via endogenous search frictions (Duffie, Gărbăneanu and Pedersen 2005), constitute the key economic force in our model.

In today’s international monetary system, an appeal of issuing debt denominated in dollars is the large and liquid nature of the dollar money market that facilitates dollar settlement.¹ All borrowers worldwide can see that there is a significant number of investors that own a substantial amount of dollar-denominated, short-term money market instruments such as Treasury bills, repos, or high-grade bank and firm debt. As a result, issuers benefit from being able to easily trade their revenue streams with these investors to acquire the assets necessary to settle their debt obligations.

The ease of settling obligations in dollars incentivizes borrowers to choose dollars as the denomination of the debt contract at issuance. As more dollar debt is issued, some of this issuance is sufficiently safe to expand the supply of dollar-denominated money market assets that can be used by other borrowers to extinguish their own debt. Some debt issuance, on

¹Dollar dominance has many features including dollar-denominated debt outstanding in the world being outsized relative to the wealth or GDP share of the United States (see Cetorelli and Goldberg 2012; Bruno and Shin 2015b; McCauley, McGuire and Sushko 2015; Maggiori, Neiman and Schreger 2020).
the other hand, may not be safe enough to act as settlement instruments themselves, but they still expand the demand for future settlement and therefore the returns to safe issuance. Together, dollar debt in these various forms begets more dollar debt, bootstrapping itself, leading to currency dominance.

The first dominant currency, the Dutch florin, is a useful example for highlighting the central way in which asset market liquidity benefits debt issuers. During this period, Spain was the largest and wealthiest economy with the highest volumes of trade, and the Spanish *real de ocho* ("pieces of eight") was the most common coin in the world. However, the quantity and quality of these coins, like all other metallic coins, varied across time and space, limiting their settlement benefits. The Bank of Amsterdam harnessed a financial technology to create a new currency called the "bank florin" as a denomination that existed purely on the Bank’s ledgers (and hence was not subject to frictions associated with transport, insurance, and degradation), and which could be used to settle obligations via free account transfers. Parties around the world, even those not transacting directly with Amsterdam, typically chose florin denomination over the currency of the largest economy at the time (Spain), reflecting the value of the florin’s superior liquidity coming from its ledger-based settlement technology.\(^2\)

We formalize this liquidity force for dominance by modeling a financial demand for liquidity, similar to Holmström and Tirole (1998), whereby assets are needed to settle debt obligations. At the same time, asset market liquidity depends endogenously via search frictions on the quantity of available liquid bonds, as in Duffie, Gărleanu and Pedersen (2005). The key decision we study is that of a private borrower—a firm, for short—whose revenues are in a home currency who must choose the denomination of its debt in either the home or foreign currency. Choosing to denominate in foreign currency entails costs associated with currency mismatch in the firm’s revenues and liabilities. Firms weigh this cost against the two benefits of issuing in a more liquid foreign currency: the ease of settling its own obligations and the convenience yield that accrues for issuing safe debt that can be used as a means of settlement by others, which provides more favorable borrowing terms.

First, by generating both benefits to issuance, our theory explains why empirically, there is a wide variety of borrowers that issue debt in the dominant currency in the cross-section. In today’s dollar world, some international firms have low credit ratings and hence

\(^2\)The florin’s international dominance was evident in the fact that all foreign exchange rates were quoted in relation to it, and in certain places such as Russia, only the exchange rate relative to Amsterdam was quoted until 1763 (Van Dillen, 1934, p. 105). Moreover, the trade between England and Russia was conducted exclusively through payments in Amsterdam, reflecting how the “[Bank of Amsterdam was] the clearinghouse of world trade’ (De Vries and Van der Woude, 1997, p. 87, 131).
their bonds are not sufficiently money-like to be used as settlement liquidity by other parties, but these borrowers still benefit from the settlement liquidity in the dollar money market. Other borrowers, say a safe international issuer such as the German government-backed supranational bank KfW, likely do not have as many settlement needs, but nonetheless issue dollar debt to capture the convenience yield. Historically as well, we observe a wide range of firms and sovereigns issuing pound-denominated debt in London, some of which was safe and some other which was prone to default. In our model, the issuance by one type of borrower raises the benefits of issuance for the other such that liquidity supply and liquidity demand are complementary, endogenously reinforcing the dominance equilibrium.

Second, our model emphasizes that large asymmetries in financial market liquidity generate a unique dominant equilibrium. Historically, this asymmetry has come from large changes in the float of safe short-term government-backed liabilities. The government backing has taken various forms, whether through physical reserve holdings or via fiscal commitments. At the Bank of Amsterdam, confidence in the City of Amsterdam’s commitment not to appropriate the specie reserves that backed the florin was crucial for enabling it to implement its ledger technology, thereby creating a large float of florin. In contrast, Spain’s history of serial defaults made it unable to provide such a commitment. Similarly, the pound sterling’s dominance was established after Dutch losses in the Fourth Anglo-Dutch War caused a collapse in confidence in Amsterdam and the British won the Napoleonic Wars. At that point, British debt became the safest in the world, backed by its fiscal capacity. In the post-Bretton Woods period, the United States government has backed a large quantity of dollar-denominated short-term debt instruments in the form of Treasury bills, with more safe float than the government bonds of alternative currencies.

Our theory clarifies that the role of economic size in generating dominance is through its impact on financial market liquidity, but that size is not itself a fundamental determinant of dominance. Increasing the size of the private sector in the non-dominant country can in fact entrench dominance, rather than leveling the field, because it may be optimal for those firms to stay in the dominant equilibrium, thereby increasing supply and liquidity in the dominant currency. In our model, as was often the case historically, the dominant currency need not be hosted by the country with the largest GDP. For example, the florin was dominant despite Amsterdam being several times smaller than Spain, and at the turn of the 20th century, the United States surpassed the British empire in economic size, but American firms continued to issue bonds in London in sterling rather than domestically in dollars. In line with our focus on asymmetry in financial market depth, the shift from sterling to the dollar only
occurred after 1913, when the US increased its issuance of government debt and the creation of the Federal Reserve system pooled different regional financial centers into a central market that helped enhance the dollar money market’s liquidity.

Third, the dual issuance benefits in our model provide novel predictions for the impact of increasing the supply of safe government debt on the issuance incentives of private borrowers. We show that within a given dominant equilibrium, for a reasonable set of parameters, safe government debt issuance crowds out safe private debt issuance, as has been empirically documented in Krishnamurthy and Vissing-Jorgensen (2012), while crowding in risky debt issuance. This result contrasts with other work that does not distinguish between different types of private debt or focuses solely on safe debt, and it allows us to explain the wide cross-section of firm issuance in the data, particularly by borrowers that do not earn a convenience yield on their debt.

Fourth, our model shows that the country hosting the dominant currency has a greater capacity and incentive to invest in institutions that expand fiscal capacity and financial market liquidity, thereby generating an additional source of complementarity that entrenches dominance and leads to the persistence of regimes. Consider a costly action that expands a country’s fiscal backing and allows for the issuance of more safe assets. The convenience yield from debt issuance, which is seigniorage revenue for the government, is higher in the dominant currency. In addition, the incentive to undertake this action is lower for the non-dominant country since the marginal gain in seigniorage revenues is lower and additionally, liquidity expansion in the non-dominant currency does not benefit firms that have already chosen the dominant currency. It is therefore beneficial for the dominant country to invest more in costly actions that create more fiscal backing—for example, by investing in the country’s military capacity to capture more resources. This expansion allows for more debt issuance, which endogenously attracts more firms to the dominant currency. As a result, dominance facilitates investments that further entrench the dominant regime. Historically, dominant country investments, for example in Amsterdam and the UK, have been directed towards financial and legal institutions as well as military capacity.

Fifth, international trade activity interacts with debt issuance by allowing the possibility that firms receive their revenues in the foreign currency (through trade invoicing). This reduction in the costs of dominant currency debt denomination reinforces dominance in a complementarity first modeled by Gopinath and Stein (2021) and Chahrour and Valchev (2022). As in those models, we also rationalize the joint observation of trade and finance in a dominant currency. Our theory, however, highlights why the volume of trade need not be
the primary driver of the dominant currency paradigm, as we have seen in previous historical episodes. In addition, it explains why the volume of foreign currency financial flows may be far larger than the volume of flows in goods, as it is in today's dollar world.

Sixth, our theory sheds light on normative aspects of liquidity provision arrangements in the international monetary system. The decision to denominate debt in the dominant currency carries a positive externality, as it improves market thickness for all other borrowers and lenders. As a result, the competitive equilibrium is not efficient: a global planner will want to subsidize even more entry in the dominant currency. Socially beneficial coordination can be implemented with formal international arrangements, like the Bretton Woods system, in which the commitment devices backing a country's monetary liabilities (such as gold) are concentrated in a single country, while all other countries have no formal need for gold.

Finally, we extend the model to include default risk and liquidity provision policy tools. At least since the Bretton Woods arrangement, the US has committed to state-contingent expansions of dollar liquidity via central bank swap lines. The Federal Reserve provided these swap lines in several recent crises when global banks and foreign central banks exhibited large demands for dollars to settle their firms' liabilities. Our model shows that such expansions enhance dominance. We introduce aggregate shocks to firms' liquidity demand and allow the government to tailor liquidity supply to depend on the aggregate state. State-contingent policies that generate positive covariance between liquidity supply and liquidity demand, such as central bank swap lines during crises, make the dominant currency more attractive. By the same token, however, if liquidity supply falls in a high liquidity demand state—negative covariance—this can undercut a currency's appeal. This latter point helps explain why the euro has lost reserve currency ground to the dollar after the 2008 financial crisis (Maggiori et al., 2020), when Euro-area sovereign debt downgrades were followed by a sharp contraction in the international use of the euro.

**Related Literature.** Our paper most directly relates to work that explains the emergence of a dominant unit of account in financial contracts. Doepke and Schneider (2017) presents a model where agents in a chain of production choose the denomination of contracts for goods purchases and sales. In this setting, it is efficient to avoid asset-liability mismatch that could lead to costly default by coordinating on a single denomination of these contracts. Furthermore, if there is a sufficient quantity of government debt that agents hold as assets, then the unit of denomination of this debt will be chosen as the unit in private contracts. In the international setting, Gopinath and Stein (2021) and Chahrour and Valchev (2022) consider a
model where agents simultaneously choose the unit of denomination of their revenues (trade invoicing) and of their debts (liability invoicing). Given the need to finance imported goods, coordinating assets and liabilities in the same unit of denomination is optimal.

The cost side in the debt denomination decision, i.e. currency mismatch, is the same in our paper as in those cited above. However, our paper highlights new channels for the benefits of denominated financial contracts in the dominant currency stemming from ease of settlement in financial assets. There is an empirical literature showing that a wide variety of firms, including non-exporters, choose foreign currency debt denomination and are rarely fully hedged against exchange rate fluctuations (Aguiar, 2005; Verner and Gyöngyösi, 2020; Adams and Verdelhan, 2022). Our model of the benefits of liquidity, which accrue to all firms, rationalizes this wide cross-section of issuance and the phenomenon of firms choosing foreign currency exposure in equilibrium, both historically and today.

The papers described above are models of the unit of account. There is a broader literature on the role of money as both a unit of account and medium of exchange. In our paper, financial assets are used as a liquid medium that provides settlement benefits, leading to the denomination of this medium to be chosen by issuers as the unit of account for their debt contracts, which further contributes to the liquidity of the medium of exchange in that unit of account. Thus, the government deepening the asset pool that trades in a given unit of account will act as a spark for currency dominance by raising that medium’s liquidity benefit of settlement and igniting these mutually reinforcing dynamics.

The endogenous emergence of dominance does not depend on payoff risk in a denomination or investor preferences. Maggiori (2017) and Gourinchas and Rey (2022) present models where the rest-of-the-world is effectively more risk-averse than the US, generating a world demand for safe (dollar) bonds and a low interest rate on US (dollar) bonds. In equilibrium, the center country (the US) harvests a premium by issuing (dollar) bonds to the rest-of-the-world. While our model also explains why firms in the center country issue debt in the dominant currency, it additionally explains why entities outside the center country have also historically been large issuers of debt in the reserve currency.

The model also sheds light on the history of reserve currencies, incentives of the sovereign, and the architecture of the international monetary system. Work on the nature of

3For example, Kiyotaki and Wright (1989) models the endogenous rise of fiat money as a medium exchange, and in the international context Matsuyama et al. (1993) as a vehicle currency.

4Hassan (2013) presents a model where the large size of the US endogenously leads to a low interest rate on dollar bonds. Eren and Malamud (2022) presents empirical evidence showing that over longer horizons the dollar depreciates after negative shocks, and this can make it optimal for firms to issues debt in dollars as a hedge against the downturns.
the international monetary system, which we relate to, includes Nurske (1944), Obstfeld et al. (1995), Tirole (2002), Gourinchas and Rey (2007), Farhi and Maggiori (2018), and He et al. (2019). Quinn and Roberds (2014b,a) and Bolt et al. (2023) describe the experience of the Dutch florin in the 17th and 18th centuries. Similarly, King (1972), Dickson (1967), Lindert (1969), Eichengreen (2008), and Kynaston (2015a,b) examine British pound dominance in the early 19th and early 20th centuries. Krugman (1984), Frankel (1992), Bruno and Shin (2015a,b), Ivashina et al. (2015), Bahaj and Reis (2020), Maggiori et al. (2020), Correa et al. (2022), and Jiang et al. (2022) examine different aspects of the current dollar-dominant regime in international finance. Our theory supports the narrative of Eichengreen (2012) and Eichengreen et al. (2017) that throughout many historical episodes, financial development in the center country that deepened financial markets played crucial roles in supporting currency dominance. In addition, we highlight how financial market depth is complementary to investments in fiscal capacity.

Our paper also belongs to the literature on the role of safe assets in the economy. Theoretical work in this area explores the macroeconomic and asset pricing implications of safe asset shortages (Holmström and Tirole 1998; Caballero et al. 2008; Caballero and Krishnamurthy 2009), and fiscal limitations in the creation of safe assets (Farhi et al., 2011; Obstfeld, 2012). There is also an empirical literature documenting how the supply of safe impacts asset prices and quantities (Krishnamurthy and Vissing-Jorgensen 2012; Gorton et al. 2012; Greenwood et al. 2015). Our model shows that the supply and denomination of safe assets affects the determination of the dominant currency and the convenience yields of safe assets denominated in that currency. Dollar safe assets carry a convenience yield (Du et al., 2018; Jiang et al., 2021), and dollar safe assets are held as reserve assets by central banks around the world (Ilzetzki et al., 2019). Our model predicts that the growing accumulation of dollar reserves by central banks around the world reflects these institutions holding the assets necessary to facilitate liquidity provision in the denomination of firms’ financial obligations.\(^5\)

We model liquidity using the search framework of Duffie et al. (2005). Our theory features increasing returns to scale in search which, as emphasized by Weill (2020), is a well-supported characterization of financial markets. Other literature featuring increasing

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\(^5\)One can also interpret our model to shed light on the dominance of exchange rate pegs to the dollar and foreign central bank accumulation of dollar reserves (Ilzetzki et al., 2019) via this logic. By pegging the exchange rate, the central bank reduces currency mismatch costs to local borrowers, while enabling them to benefit from issuing in the liquid dollar market. But to maintain the peg, the central bank accumulates dollar assets and trades these dollars to local firms for their goods’ revenues when needed. The local firms then use the dollars to settle their dollar liabilities. The demand for these reserve assets coming from settlement needs generates a convenience yield on both public and private safe dollar claims.
returns to scale in financial markets includes Pagano (1989), Duffie et al. (2007), Garleanu and Pedersen (2007), Vayanos and Wang (2007), Vayanos and Weill (2008), Weill (2008), Lagos and Rocheteau (2009), Shen et al. (2018), Sambalaibat (2022), Geromichalos and Herrenbrueck (2022), and Geromichalos et al. (2023). Further, we connect to work highlighting the importance of search frictions in sovereign debt pricing and exchange rate determination, including Chaumont (2020), Moretti (2020), Bianchi et al. (2021), and Passadore and Xu (2022). We trace out the implications of these search-based illiquidity frictions for the corporate financing decisions of firms around the world.

2 A Model of Liquidity and Debt Denomination

We consider a three-period \((t = t_0, t_1, t_2)\) environment with two countries, indexed by \(j \in \{A, B\}\). In each country, there is government-backed risk-free debt of mass \(G_j\) denominated in units of the local currency \(j\), which can be traded to meet the liquidity needs of agents. The mass \(G_j\) includes all money-like instruments that the government has committed to guarantee.

There are also entrepreneurs who run firms that issue safe debt (e.g., bills of exchange or corporate bonds), in mass \(F_j\). These private borrowers make a choice of denominating the debt in their home currency or in the foreign currency. Firms have liquidity needs in a manner similar to Holmström and Tirole (1998). Asset trading occurs in a secondary market with endogenous trading frictions, as in Duffie et al. (2005). Finally, there is a continuum of homogeneous risk-neutral investors that buy the debt of firms and governments.

2.1 Within-Country Environment

We start by developing the model within a given country, where we consider the choice to issue debt of private-sector borrowers. In Section 2.2, we turn to the full case where firms may choose the currency in which to issue debt and characterize the general equilibrium.

2.1.1 Debt Issuance at \(t_0\)

There is a mass of entrepreneurs \(F_j\) in each country \(j\). Each entrepreneur owns a firm that can borrow to invest in a project at \(t_0\). The project will generate profits of one at a stochastic time, either \(t_1\) or \(t_2\). The investment has a cost \(\beta^2\), which is incurred at \(t_0\). At \(t_0\), the entrepreneur can raise funds for the investment by selling debt with face value of
one maturing at \( t_2 \), which will be repaid using the future profits. As will become clear, the model is set up so that borrowing and investment is always profitable for the entrepreneur.

The preference of a given entrepreneur \( i \) is to maximize:

\[
u^F_{i,j} = c_0 + \beta c_1 + \beta^2 c_2, \quad c_t \geq 0, \quad \beta < 1.
\]

At \( t_0 \), the government also issues a quantity \( G_j \) of government-backed securities. We assume that both government and private bonds are safe (i.e., there is no default risk, as we elaborate on in Section 2.3), and that they have the same liquidity properties—hence, they are priced the same in equilibrium. We consider breaking this symmetry later in the analysis. We also consider the case of risky bonds later in the analysis. For now, as the bonds are identical they have the same endogenous price \( P_{0,j} \).

There is a mass \( I_j \) of investors who have sufficiently large endowments to purchase bonds issued by the government and firms at \( t_0 \). The investors are risk neutral with preferences

\[
u^I_{i,j} = c_0 + \beta c_1 + \beta^2 c_2, \quad c_t \geq 0.
\]

Each investor potentially owns one bond, and bonds are indivisible. The total mass of bonds is \( G_j + F_j \). Define the total mass of bondholders to be

\[
m_{I,j} = G_j + F_j \leq I_j,
\]

where the last inequality is a restriction on parameters. That is, we assume there are enough investors to purchase all of the bonds at \( t_0 \).

### 2.1.2 Money Market Settlement With Search Frictions

The liquidity need in the model arises if the entrepreneur’s profits arrive early at \( t_1 \) while his debt is due at \( t_2 \). In this case, his debt and profits streams are mismatched in time. The probability of receiving profits early is \( \phi \). The mass of liquidity-demanding firms is the total number of early firms, which, by the law of large numbers, is

\[
m_{F,j} = \phi F_j.
\]

With the possibility of early revenues, it may be beneficial for firms to obtain a financial asset at \( t_1 \) because such an asset would allow them to have a savings vehicle to extinguish their \( t_2 \) debt obligations. The financial assets that firms seek are the bonds issued by the
government and other firms at date $t_0$. We model obtaining financial assets from the money market as search and matching.

Firms with early profits may trade with investors at $t_1$ to obtain the financial asset (i.e., a bond) for settlement at $t_2$. There are gains from trade in a meeting. We assume that if the firm does not trade with the investor, then it keeps its profits until $t_2$ and uses the profits to settle its debt. The effective return on keeping its profits is therefore zero. However, since investors discount the future at rate $\beta < 1$, an investor who owns a bond is willing to sell the bond as long as he receives at least a quantity $\beta$ of goods that he consumes at $t_1$. The gains from trade in a match between investor and firm is therefore $1 - \beta$. We assume the firm receives a fraction $\eta$ of this surplus and the investor keeps the remaining $1 - \eta$ share.

We now describe the search market at $t_1$. We posit a matching function such that the number of meetings between liquidity demanders (firms) and liquidity suppliers (date $t_0$ investors) is

$$n_j = \lambda_j m_{F,j} \theta m_{I,j} \theta, \quad \frac{1}{2} < \theta \leq 1.$$  \hspace{1cm} (5)

Here $\lambda_j > 0$ captures the overall degree of liquidity of the money market. In the continuous-time asset trading model of Duffie et al. (2005, henceforth DGP), $\lambda_j$ corresponds to the Poisson probability that a given agent (say, a firm) will meet another agent (say, an investor). Given Poisson meeting rates, $\theta = 1$ so that the total number of matches is proportional to the masses of both firms and investors (Duffie et al., 2018).

The key property of this matching function is increasing returns to scale, corresponding to $\theta > \frac{1}{2}$. If the masses of both firms and bond-holding investors double, the number of matches more than doubles. Thus the search model embeds a thick-market liquidity externality as in Diamond (1982). This liquidity externality is at the heart of our theory.

Given the matching function, the endogenous two-sided meeting probabilities are:

$$\alpha_{F,j} = \frac{n_j}{m_{F,j}} = \lambda_j m_{F,j}^{\theta-1} m_{I,j}^\theta, \quad \alpha_{I,j} = \frac{n_j}{m_{I,j}} = \lambda_j m_{F,j}^\theta m_{I,j}^{\theta-1}.$$  \hspace{1cm} (6)

Here $\alpha_{F,j}$ is the probability of a firm meeting a bond seller (date $t_0$ investor in bonds) at time $t_1$, and $\alpha_{I,j}$ is the probability that the bond seller meets a firm. The key economic force that the modeling captures is that the trade to obtain assets for settlement is frictional, and that a greater outstanding quantity of bonds makes obtaining this liquidity easier (higher $\alpha_{F,j}$). Figure 1 provides a timeline of the debt market.

The date $t_1$ liquidity market as described is an over-the-counter (OTC) bond market.
Figure 1: **Timeline of debt issuance and demand for settlement**

<table>
<thead>
<tr>
<th>$t_0$</th>
<th>$t_1$: Search</th>
<th>$t_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F,G$ issue bonds for $P_0$</td>
<td>$m_F$ firms with early profits (mass $\phi F$) can match with $m_I$ potential sellers with probability $\alpha_F$</td>
<td>Bonds mature</td>
</tr>
</tbody>
</table>

**Notes:** This figure presents a schematic representation of timing in the model.

as in Duffie et al. (2005), where firms trade goods with investors for their one-period bonds. However, we do not take a stand on the market structure of this trade, and it is likely that this structure has varied across history, as we describe in Section 2.3.

### 2.1.3 Firm Issuance Decision and Asset Market Equilibrium

We formally present the entrepreneur’s decision problem. The entrepreneur makes an issuance decision at date $t_0$. Denote $D_i$ as an indicator function that takes the value one if the firm issues debt to invest and zero if the firm does not. The firm decides at date $t_1$ to trade for a bond or not. Denote $T_i$ as an indicator function that reflects the decision to trade. Then the entrepreneur’s problem is:

$$\max_{D_i, T_i} E[c_0 + \beta c_1 + \beta^2 c_2].$$

The structure we have posited imposes several constraints on this problem. Consumption at date $t_0$ is $c_0 = D_i(P_{0,j} - \beta^2)$. Consumption at date $t_1$ is $c_1 = D_i T_i \eta (1 - \beta)$ if the profits arrive early and the firm matches with a counterparty in the asset market, and zero otherwise. Consumption at date $t_2$ is always zero in equilibrium, $c_2 = 0$. Then, as long as $P_{0,j} \geq \beta^2$ and $1 - \beta > 0$, the solution is to set $D_i = 1$ and $T_i = 1$.

We solve for $P_{0,j}$ backwards. Consider the market at date $t_1$ first. If a match occurs, the total surplus is $1 - \beta$, of which a bond seller obtains $(1 - \eta)(1 - \beta)$. We assume that the date $t_0$ bond market is Walrasian. Each investor can bid for exactly one bond at date $t_0$. If an investor purchases a bond at $t_0$, the investor either resells the bond at date $t_1$ to earn $\beta + (1 - \eta)(1 - \beta)$, or the investor holds the bond to maturity. Thus the investor’s valuation of the bond at $t_0$ is:

$$P_{0,j} = \frac{\alpha_{I,j}\beta [\beta + (1 - \eta)(1 - \beta)]}{P(\text{Matched}) \times \text{PV of Sales Price}} + \frac{(1 - \alpha_{I,j})\beta^2}{P(\text{Not Matched}) \times \text{PV of 1}}.$$  

11
or rewriting:

\[
P_{0,j} = \beta^2 + \alpha_{I,j} \beta(1 - \eta)(1 - \beta).
\]  

(9)

Since \(1 - \eta > 0\), we have that \(P_{0,j} > \beta^2\), so that \(D_i = 1\) in the firm’s issuance problem.

We define the wedge \(P_{0,j} - \beta^2\) as a convenience yield on bonds issued at \(t_0\). Consider the pricing of a completely illiquid bond, which in our model is one for which \(\alpha_{I,j} = 0\). This bond will be priced at \(\beta^2\). The government and private firm bonds in our model are priced at \(P_{0,j} > \beta^2\) because they offer settlement liquidity to firms at date \(t_1\). The convenience yield increases in the match probability (\(\alpha_{I,j}\)) and the surplus gained from the match ((1 - \(\eta\))(1 - \(\beta\))).

Finally, consider the entrepreneur’s expected utility from bond issuance at date \(t_0\) at an endogenous price \(P_{0,j}\):

\[
u_{I,j}^F = P_{0,j} - \beta^2 + \beta \phi \alpha_{F,j} \times \eta(1 - \beta).
\]  

(10)

The first two terms in this objective reflects the benefit from selling bonds at a high price at \(t_0\) minus the cost of investment, which is the convenience yield. The second term reflects the benefit of settlement liquidity at \(t_1\). The firm is early with probability \(\phi\) and obtains the needed liquidity with probability \(\alpha_{F,j}\). The share of the surplus in the trade that the firm receives is \(\eta(1 - \beta)\), and the firm discounts the future at \(\beta\).

Given equilibrium bond prices, we can rewrite (10) as:

\[
u_{I,j}^F = \lambda_j \beta(1 - \beta) (m_{F,j}m_{I,j})^{\theta-1} [(1 - \eta)m_{F,j} + \phi \eta m_{I,j}].
\]  

(11)

The two additive terms in this expression reflect the two ways in which firms benefit from money market liquidity: the first term reflects the benefit of capturing convenience yields on the firms’ initial issuance, which is increasing in \(m_{F,j}\), the mass of firms demanding settlement liquidity, and in \(1 - \eta\), the surplus share accruing to the owner of the bond if a trade happens. The second term reflects the benefit from a high probability of being able to find a match in the date \(t_1\) money market, which is increasing in the mass of available bonds \(m_{I,j}\), in the surplus share \(\eta\) going to the firm needing settlement, and in \(\phi\), the probability that the firm needs settlement.

In equation (11), both benefits accrue to the same issuers because firms are homogeneous. In practice, certain issuers can be best thought of as pure liquidity providers who harvest the convenience yield but do not have large transaction needs. On the other hand,
different issuers may not be able to capture convenience yields because their debt does not offer sufficient liquidity benefits (e.g., because of default risk), but may still value settlement liquidity—and hence may be best thought of as pure liquidity demanders. In Section 3.5, we consider an extension in which we separate these two roles in the cross-section of firms.

From now on, we set \( \eta = \frac{1}{2} \), as we do not explicitly model the bargaining process and \( \eta \) plays no part in the analysis. A firm’s expected utility from debt issuance is therefore:

\[
\begin{align*}
  u^F_{i,j} &= \frac{1}{2} \lambda_j \beta (1 - \beta) (m_{F,j}m_{I,j})^{\theta-1} [m_{F,j} + \phi m_{I,j}].
\end{align*}
\] (12)

\[2.2 \text{ International Equilibrium Conditions}\]

We next describe the international equilibrium. The two countries, \( j = A, B \), have fundamentals \( \{G_j, \lambda_j, F_j\} \). Firms earn revenues in domestic currency and choose the denomination of bonds, either domestic or foreign. We assume that the government only issues bonds in its domestic currency.

We assume that there is a fixed cost of issuing in a foreign currency proportional to \( K_i \), which is heterogeneous across firms. In Appendix Section A.1, we provide a microfoundation in terms of costs associated with running a balance sheet currency mismatch.\(^6\) We assume that \( K_i \) is distributed on \([K, \infty)\) with cumulative distribution function \( H(K_i) \), and corresponding density \( h(K_i) = H'(K_i) \). This density is identical in the two countries.

We let the set of buyer and seller masses be \( \mathcal{M} = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B}) \). We can compute expected utility for the entrepreneurs in the two countries and for each of the possible denomination choices. These four expressions are as follows:

1. **Expected utility of entrepreneur in country B issuing in foreign currency (A):**

   \[
   U_{B \rightarrow A}(\mathcal{M}, K_i) \equiv \frac{\beta(1 - \beta)}{2} \left[ \lambda_A (m_{F,A}m_{I,A})^{\theta-1} [m_{F,A} + \phi m_{I,A}] - K_i \right].
   \] (13)

2. **Expected utility of entrepreneur in country B issuing in home currency (B):**

   \[
   U_{B \rightarrow B}(\mathcal{M}) \equiv \frac{\beta(1 - \beta)}{2} \lambda_B (m_{F,B}m_{I,B})^{\theta-1} [m_{F,B} + \phi m_{I,B}].
   \] (14)

\(^6\)One can also alternatively think of the cost \( K_i \) as capturing the costs of hedging foreign currency exposures. In addition, \( K_i \) might also incorporate fixed costs of finding a bank and underwriters of bonds in the foreign currency, providing information so that foreign investors can assess the firms’ risks, and so on.
3. Expected utility of entrepreneur in country A issuing in foreign currency (B):

\[
U_{A\rightarrow B}(\mathcal{M}, K_i) \equiv \frac{\beta(1-\beta)}{2} \left[ \lambda_B (m_{F,B} m_{I,B})^{\theta-1} [m_{F,B} + \phi m_{I,B}] - K_i \right].
\]  

(15)

4. Expected utility of entrepreneur in country A issuing in home currency (A):

\[
U_{A\rightarrow A}(\mathcal{M}) \equiv \frac{\beta(1-\beta)}{2} \lambda_A (m_{F,A} m_{I,A})^{\theta-1} [m_{F,A} + \phi m_{I,A}].
\]  

(16)

We index denomination choice by \(D_{i,j}\), where \(D_{i,j} = 1\) if firm \(i\) in country \(j\) issues in foreign currency, and \(D_{i,j} = 0\) otherwise. Each firm chooses its debt denomination optimally by comparing the expected utility functions given above:\(^7\)

\[
D_{i,j} = \begin{cases} 
1 & \text{if } U_{j\rightarrow j'}(\mathcal{M}, K_i) > U_{j\rightarrow j}(\mathcal{M}), \\
0 & \text{if } U_{j\rightarrow j'}(\mathcal{M}, K_i) \leq U_{j\rightarrow j}(\mathcal{M}).
\end{cases}
\]

(17)

We then have three results. First, since \(U_{j\rightarrow j'}\) is monotonically decreasing in \(K_i\), we obtain:

**Lemma 1.** Consider firms \(i\) and \(i'\) in country \(j\), where \(K_i < K_{i'}\). If it is optimal for firm \(i\) to issue in foreign currency \(j' \neq j\), then it is also optimal for firm \(i\) to issue in foreign currency \(j'\).

Next, notice that the expressions for firm utility in country A have the same terms as the expressions for firms in country B. As a result:

**Lemma 2.** Suppose that there is a positive mass of firms in \(j\) that find it optimal to issue in currency \(j'\). Then, no firms in \(j'\) will issue in currency \(j\).

In other words, if some firms in \(B\) choose to pay a cost to issue in country \(A\), then a firm in \(A\) for which there is no cost to issue in \(A\) will choose to only issue in \(A\), and hence no firm from \(A\) issues in \(B\). Together, these two lemmas imply that optimal firm denomination choices must have a threshold structure, which we formalize as follows.

---

\(^7\)In writing a \(B\) firm’s problem as one of comparing the expressions for utility to issuing in \(A\) to that of \(B\), we require that conditional on a currency choice the optimal choices over issuance and settlement are as described in the previous within-country setting. With two currencies, there is another action that is feasible: a \(B\) firm could issue in currency \(B\), then at date \(t_1\) the early-firm converts its goods into currency \(A\) and searches for a bond in \(A\), and finally at \(t_2\), the firm converts its bond proceeds back into \(B\) and uses these proceeds for settlement. We assume that these two further foreign exchange transactions expose the firm to currency risk and thus the firm incurs additional costs. In this case, while this action is feasible, it is never optimal.
Lemma 3. A necessary condition for a collection of firm denominations choices $D_{i,j}$ to be consistent with firm optimality is that it must take the following threshold form:

$$
D_{i,j'} = \begin{cases} 
1 & \text{if } K_i < \bar{K}, \\
0 & \text{if } K_i \geq \bar{K},
\end{cases} \quad D_{i,j} = 0. \tag{18}
$$

In the lemma above, $\bar{K}$ corresponds to the threshold cost below which firms choose to issue in foreign currency: it is a scalar that provides a sufficient statistic summarizing the entirety of the set of all firms’ denomination choices. A corollary is that the masses $\mathcal{M}$ can themselves be represented as functions of the threshold cost $\bar{K}$: $\mathcal{M} = \mathcal{M}(\bar{K})$. Further, we introduce the following notation for the expected utilities of the threshold firm (for which $K_i = \bar{K}$):

$$
\bar{U}_{j' \to j} (\bar{K}) \equiv U_{j' \to j}(\mathcal{M}(\bar{K})), \quad \bar{U}_{j' \to j'} (\bar{K}) \equiv U_{j' \to j'}(\mathcal{M}(\bar{K})). \tag{19}
$$

We denote the equilibrium value of $\bar{K}$ as $\hat{K}$. This is the cost which sets the marginal firm indifferent between the two currencies, equalizing the two expressions in equation $\tag{19}$.\(^8\)

Given the threshold structure for firms strategies, we can formally define equilibria.

**Definition 1.** An equilibrium is a collection of an endogenous threshold $\hat{K}$, exogenous parameters $\Theta = (F_A, F_B, G_A, G_B, \lambda_A, \lambda_B, \phi, \theta)$, an exogenous firm size distribution $H(K)$, and endogenous masses $\mathcal{M} = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$, satisfying:

1. (Market clearing) Given $\hat{K}$, the masses $\mathcal{M}$ satisfy:

$$
m_{F,j} = \phi \left[ F_j + H(\hat{K})F_{j'} \right], \quad m_{F,j'} = \phi \left( 1 - H(\hat{K}) \right) F_{j'}, \tag{20}
$$

and,

$$
m_{I,j} = G_j + F_j + H(\hat{K})F_{j'}, \quad m_{I,j'} = G_{j'} + \left( 1 - H(\hat{K}) \right) F_{j'}. \tag{21}
$$

2. (Firm optimality) Given the masses $\mathcal{M}$, the threshold is optimal: firms in $j'$ with $K_i < \hat{K}$ find it optimal to issue in currency $j$, while all other firms optimally issue in their own currency, so that

$$
\begin{cases} 
\bar{U}_{j' \to j} (\hat{K}) = \bar{U}_{j' \to j'} (\hat{K}) & \text{for } \hat{K} > K, \\
\bar{U}_{j' \to j} (\hat{K}) \leq \bar{U}_{j' \to j'} (\hat{K}) & \text{for } \hat{K} = K. 
\end{cases} \tag{22}
$$

\(^8\)Alternatively, by complementary slackness, equilibria at the lower boundary ($\hat{K} = K$) are also valid if $\bar{U}_{j' \to j} (K) \leq \bar{U}_{j' \to j'} (K)$. 

15
Further, throughout the analysis, we will examine the stability properties of the equilibria specified in the above definition, whenever the model features multiple equilibria. We formally specify our notion of stability using the following criterion.

**Definition 2.** Consider an underlying dynamical system through which an out-of-equilibrium system converges to equilibrium (a tatonnement process) of the form \( \partial_{\tau} \hat{K} = \delta \left[ U_j(\hat{K}) - U_j'(\hat{K}) \right] \) for \( \delta > 0 \) and where \( \tau \) indexes a mass of continuous sub-periods within time \( t_0 \). An equilibrium featuring the endogenous threshold \( \hat{K} \) is said to be stable if it fulfills the following condition:

1. (Equilibrium stability) There exists an \( \varepsilon > 0 \) such that any trajectory beginning in the neighborhood \([\hat{K} - \varepsilon, \hat{K} + \varepsilon]\) converges to \( \hat{K} \).

### 2.3 Discussion of Modeling Choices

We further explain some of the modeling choices that we have made in this section.

1. **Rollover risk vs. saving:** The liquidity trade in our model at \( t_1 \) is firm “saving”, but this is more for simplicity than it is substantive. The key component is that the firm trades its goods for a one-period bond. Consider a variant of the model in which there is a date \( t_3 \), and the firm’s liquidity need stems from rollover risk. The timing mismatch in that problem is that with some probability, the firm will receive the goods at \( t_3 \) rather than \( t_2 \) while the debt is due at \( t_2 \). The firm will then want to trade its future revenues for bonds at \( t_1 \). If there are more bonds available, this trade will be less frictional and it will be less costly to rollover the debt. Again, the key economics our model captures is in linking liquidity with bond supply.

2. **Interpretation of \( G \).** While we define \( G_j \) as the supply of safe government bonds, it should be more broadly interpreted as the supply of money market instruments that depends on the government’s commitment to maintain their value. Throughout history, government commitment has taken various forms: either purely fiscal, or through physical holdings of precious metals such as in a gold standard. For example, in the context of Amsterdam, the relevant government-backed debt supply is not the sovereign debt, but rather the specie-backed bank florin. The relevant government commitment for firms was the City of Amsterdam’s promise not to appropriate the underlying specie. When confidence in this commitment collapsed, as it did in 1795, there was a drop in \( G \) even though ultimately the default rate on florin was low. The
relevance of the government commitment is also why Spain, despite having a large nominal debt in the 18th century, had low $G$. Its history of default reduced its ability to credibly commit to supporting a float of safe and liquid assets. In the modern context commitment generally takes the form of fiscal backing. We model government default risk in Section 4.5.

3. **Institutional structure of the money market at $t_1$:** The asset trade between investors and firms, in which investors that have bought safe debt at $t_0$ provide liquidity at $t_1$, often takes the form of financial intermediation. For example, in Amsterdam, a $t_0$ investment took the form of traders depositing their specie at the Bank to receive a florin deposit (Quinn and Roberds, 2014a). These traders would pay a fee to the Bank in this transaction and would therefore receive less florins than the value of the specie, as in an overcollateralized repo transaction. The traders would then lend florins to a merchant at $t_1$ who needed florins for settlement, earning interest or a fee in this transaction. From the standpoint of the model, the repo technology and specie are “liquidity supply” and the merchant is the “liquidity demander” at $t_1$.

These labels also apply in thinking about a banking arrangement or modern money market and repo. The “liquidity suppliers” ($m_I$ in the model) are banks that own the bonds as an asset and issue deposits to “liquidity demanding” firms ($m_F$) at $t_1$, which the firms then use to settle their $t_2$ debt. Alternatively, financial firms use Treasury securities as collateral in repo arrangements to obtain reserves that can then be lent to others as settlement instruments.

In the global dollar market, liquidity supply is provided by foreign central banks and global banks. These entities hold safe dollars as foreign reserves, obtained at $t_0$ in the form of Treasury securities, which they provide to firms at $t_1$. In this way, reserve holdings would tilt towards the dollar even without asymmetry in investor preferences for dollar assets—as in, for example Gourinchas and Rey (2022) or Jiang et al. (2020a).

4. **Theoretical understanding of denomination: payoff risk vs. settlement.** Currency denomination encompasses two distinct aspects: payoff risk and settlement. For example, a contract where payoffs vary with nominal exchange rates is one where denomination determines payoff risk. Our theory instead highlights the settlement aspect of denomination. To extinguish a dollar-denominated debt contract, the borrower needs to deliver dollars. As a concrete example, consider the payoffs of Swiss Franc or Canadian dollar debt. These currencies have exchange rates with similar payoff characteristics.
as the dollar: they appreciate during periods of economic turmoil. Yet, in terms of quantities, dollar debt is many orders of magnitude larger than that of Canadian or Swiss debt. Our settlement-liquidity theory predicts more private debt issuance in US dollars, whereas a payoff risk theory likely cannot. Historically, the classical gold standard era provides similar examples. Currencies like the German mark and French franc also provided a claim to the same underlying specie payoff as the British pound sterling, and yet there was much less foreign debt issuance in marks or francs relative to pounds.

5. Is search and liquidity a concern in money markets? Our model links corporate financing decisions to money market illiquidity concerns. Our analysis turns on the relative liquidity of the money markets in different currencies rather than the absolute liquidity of any one market. At a theoretical level, our model builds on a long tradition of using search to model money markets (see Kiyotaki and Wright, 1993; Lagos and Wright, 2005). Empirically, even in high-volume money markets such as the US dollar repo market, search models have been shown to capture price and quantity patterns well (Vayanos and Weill, 2008). At the macro level, during a period of global financial volatility, the dollar money market remains more liquid than the markets of many emerging and even advanced economies. In our model, these considerations drive financing decisions.

3 Currency Dominance and Denomination Incentives

Having specified the model environment and derived its equilibrium conditions in Section 2, we now turn to analyzing the properties of the resulting equilibria and examining the underlying economic forces. We outline how multiple equilibria naturally emerge as a consequence of increasing returns to scale, and how asymmetries in country fundamentals favor those equilibria that feature currency dominance. We illustrate the complementary benefits to debt issuance in the cross-section of firms, and we provide novel predictions for the impact of government debt issuance on firm issuance incentives.

3.1 Equilibria in a Special Case

In order to illustrate the forces in the model and build intuition, we begin by analyzing the simplest, fully symmetric case where fundamentals \((\lambda_j, G_j, F_j)\) are the same for both
\( j = A, B \), where we also set the foreign-currency issuance cost \( K_i \) to zero for all firms \( i \), and where restrict to \( \theta = 1 \), which yields the most analytical tractability.\(^9\) The next subsection analyzes the model in generality, relaxing all of these assumptions.

**Proposition 1 (Special case: \( K_i = 0, \theta = 1 \)).** There are three equilibria in the symmetric, \( K_i = 0, \theta = 1 \) case:

1. No firm switches to issue in the foreign currency.
2. All firms from \( B \) switch to issuing in currency \( A \).
3. All firms from \( A \) switch to issuing in currency \( B \).

We refer to equilibria in which firms in country \( B \) switch to currency \( A \) (equilibrium 2 above) as “class BA equilibria”, and to equilibria in which firms in country \( A \) switch to currency \( B \) (equilibrium 3 above) as “class AB equilibria.”

Consider each of these cases in turn:

1. No firm switches, so that the masses are symmetric across \( A \) and \( B \). As a result the utility from issuing in the home currency equals that of issuing in the foreign currency. The convenience yields on bonds in both \( A \) and \( B \) are equal:

\[
P_{0,j} - \beta^2 = \frac{\lambda_j \beta(1 - \beta)}{2}m_{F,j}.
\]

   (23)

2. All firms switch to \( A \). Starting from the no-firms-switch case, if a small mass of firms were to shift from \( B \) to \( A \), then the masses \( m_{F,A} \) and \( m_{I,A} \) would rise while the corresponding masses in \( B \) would fall. Then because of increasing returns to scale in matching, the liquidity benefit from issuing in \( A \) rises relative to that of \( B \). We can see this by examining the liquidity benefit of switching to \( A \):

\[
U_{B\to A} = \frac{\lambda_A \beta(1 - \beta)}{2} \left[ m_{F,A} + \phi m_{I,A} \right].
\]

   (24)

Equation (24) highlights the two forces that drive firm decisions. Consider the second term in brackets. A firm that needs liquidity at date \( t_1 \), which occurs with probability \( \phi \), benefits from having a larger pool of liquidity, which is linear in \( m_{I,A} \). Next consider the first term in the bracket. At date \( t_0 \), firms sell their bonds at a convenience yield

\[^9\text{In this case the distribution of costs } K_i \text{ is degenerate with support only at zero: } H(K) = 1 \text{ for } K \geq 0, \text{ and } H(K) = 0 \text{ otherwise.}\]
because these bonds are used for settlement purposes at date $t_1$. This convenience yield benefit is increasing in $m_{F,A}$, the mass of firms needing early settlement.

As the liquidity benefit of moving to $A$ rises, the liquidity benefit of remaining in $B$ falls. The result is that the equilibrium resolves with all firms switching to $A$. In this equilibrium, the convenience yield on bonds in $B$ is zero, while it is positive in $A$.

3. All firms switch to $B$. This is the symmetric case as that of (2).

The fully symmetric case illustrates the economics at work. The dominant equilibria of all firms switching to either $A$ or $B$ are the only stable ones. The equilibrium in (1) is unstable: a movement in masses to either $A$ or $B$, such as from a perturbation in $G$, shifts the benefits for all other firms in the same direction leading to the all-firms-switch equilibria.

### 3.2 Equilibria in the General Case

We now analyze the model in the general case. The following proposition characterizes the configuration of model equilibria in the general case, where we allow for unrestricted (and hence possibly asymmetric) country fundamentals, heterogeneous positive costs $K_i \in [K, \infty)$, and a degree of returns scale $\theta \in (\frac{1}{2}, 1]$.

**Proposition 2** *(Equilibrium characterization).* *The model can generate three classes of stable equilibria:*

1. **Interior equilibria featuring entry from country B firms into currency A** (class $BA$).
2. **Interior equilibria featuring entry from country A firms into currency B** (class $AB$).
3. **No-entry equilibria, in which no firm switches to foreign currency.**

*Depending on the parameter vector $\Theta$, these different classes of equilibria can all be present at the same time: hence, the model features equilibrium multiplicity. Alternatively, for particular parameter configurations, stable equilibria of only one class or of only two classes can be present.*

Proof. See Appendix Section A.2.

To understand the economic forces shaping this equilibrium configuration, we now provide a discussion and graphical analysis where, for ease of illustration, we parameterize $\theta = 1$. We also set $H(K)$ as a Pareto distribution, so that the cumulative distribution function takes the form $H(K) = 1 - \left(\frac{K}{\overline{K}}\right)^\alpha$ where $\alpha > 0$ is a shape parameter. This distribution features
several properties that are salient in the cross-section of firms. In particular, it captures the notion that most debt issuance is done by a tail of very large firms with low per-unit costs $K_i$. These large firms will be the first to sort endogenously into foreign currency issuance, and therefore small increases in the threshold $\bar{K}$ in the neighborhood of the lower boundary $\bar{K}$ will give rise to disproportionately large movements in the masses $\mathcal{M}$, as compared to similar increases in $\bar{K}$ at higher levels of entry.

In class $BA$ equilibria, firms in country $B$ play the threshold strategy described in Lemma 3, and all firms in country $A$ issue in home currency. The marginal firm in country $B$ (one for which $K_i = \bar{K}$) is indifferent between issuing in the two currencies. Therefore an interior equilibrium threshold $\hat{K}$ satisfies the following indifference condition, which is a specialization of equation (22):

$$
\lambda_A \left[ m_{F,A} + \phi m_{I,A} \right] - \hat{K} = \lambda_B \left[ m_{F,B} + \phi m_{I,B} \right].
$$

The masses of liquidity demanders (buyers) in the two currencies are

$$
m_{F,A} = \phi \left[ F_A + H(\hat{K}) F_B \right], \quad m_{F,B} = \phi \left[ 1 - H(\hat{K}) \right] F_B,
$$

while the masses of liquidity suppliers (sellers) are

$$
m_{I,A} = G_A + F_A + H(\hat{K}) F_B, \quad m_{I,B} = G_B + \left[ 1 - H(\hat{K}) \right] F_B.
$$

Figure 2a plots the curves $\bar{U}_{B \to A}$ and $\bar{U}_{B \to B}$ as functions of the threshold cost $\hat{K}$, keeping the country fundamentals ($\lambda_j, G_j, F_j$) symmetric. These curves capture the expected utility of the marginal firm ($K_i = \bar{K}$) from issuing in foreign currency or home currency. The shapes of these two curves reflect the economic forces at work. The curve $\bar{U}_{B \to B}$ is monotonically decreasing since higher values of $\hat{K}$ correspond to higher entry into currency $A$, which reduces the thickness of currency $B$ markets, lowering the utility of home currency issuance. Conversely, higher entry raises the expected utility of foreign currency issuance, which is a force pushing $\bar{U}_{B \to A}$ higher. The curve $\bar{U}_{B \to A}$ is however also subject to a second force, since as $\hat{K}$ increases, the identity of the marginal firm changes: it is now a firm with

---

10 The size distribution of firms is fat-tailed and well-described by a Pareto form (Gabaix 2011, Chaney 2018). In addition, we expect foreign currency issuance fixed costs (Maggiori et al. 2020) to give rise to a negative correlation between costs paid per unit of debt and firm size.

11 For ease of exposition, here we do not separately write out the condition for equilibria at the left boundary $\hat{K} = K$, although these are of course valid if the complementary slackness condition in the second row of equation (22) is satisfied.
higher foreign issuance cost \(K_i\). This gives rise to a linearly decreasing component of \(\bar{U}_{B\rightarrow A}\). Given the Pareto distribution of costs, there is a diminishing marginal impact of entry from increasing the threshold cost \(\hat{K}\), which mediates the relative strength of the forces impacting \(\bar{U}_{B\rightarrow A}\), giving rise to its non-monotonic (concave) shape.

The model features three equilibria of class BA in this example. Two equilibrium points (labeled 1 and 2, and occurring at \(\hat{K}_1\) and \(\hat{K}_2\), respectively) lie at the intersections of the two curves \(\bar{U}_{B\rightarrow A}\) and \(\bar{U}_{B\rightarrow B}\). At these intersections, the expected utility from issuing in A and B is equalized for the marginal firm with threshold cost \(\hat{K} = \hat{K}\). The interior equilibrium featuring low entry (point 1) is unstable while the high-entry equilibrium (point 2), featuring currency dominance, is stable. A further equilibrium point (labeled 0) is at \(\hat{K} = K\). In this equilibrium, no firms issue in foreign currency: this equilibrium is now stable (relative to the \(K_i = 0\) case) because of the presence of fixed costs \(K_i > 0\), which make \(\bar{U}_{B\rightarrow B}\) higher than \(\bar{U}_{B\rightarrow A}\) in the neighborhood of \(\hat{K} = K\).

As also seen in Proposition 1, there is a second, symmetric class of equilibria as well. These are equilibria in which firms in country A issue in foreign currency, while all firms in country B remain in home currency (class AB equilibria). For this class of equilibria, \(\bar{K}\) now characterizes the threshold strategy played by firms in country A. The analysis is analogous. Since it is now the marginal firm in country A that needs to be indifferent, the indifference condition determining an interior equilibrium threshold \(\hat{K}\) in this case is:

\[
\frac{\lambda_B [m_{F,B} + \phi m_{I,B}]}{\bar{U}_{A\rightarrow B}} - \hat{K} = \frac{\lambda_A [m_{F,A} + \phi m_{I,A}]}{\bar{U}_{A\rightarrow A}},
\]

(28)

and the masses \(\mathcal{M}\) are obtained by performing the analogous specialization of (20) and (21).

Figure 2b provides a graphical analysis by showing the curves \(\bar{U}_{A\rightarrow B}\), and \(\bar{U}_{A\rightarrow A}\) as functions of the threshold \(\hat{K}\). As in the previous case, there are two interior equilibria: a low-entry unstable one (labeled 3) and a high-entry stable one (labeled 4). The equilibrium in which no firm switches (at \(\hat{K} = K\), labeled 0) also features. This is because the no-entry equilibrium has two isomorphic representations: one as a class BA equilibrium, and one as a class AB equilibrium.\(^{12}\)

We note that despite the presence of currency dominance, our model with positive foreign issuance costs rules out winner-takes-all equilibria in which only one currency survives,
Figure 2: Characterizing equilibria in the heterogenous cost case

Notes: Panel A plots the expected utility of a firm in country B switching to issuing in foreign currency ($U_{B \rightarrow A}$), and of a firm in B issuing in home currency ($U_{B \rightarrow B}$), as a function of the threshold cost $K$. Panel B plots the expected utility of a firm in country A switching to issuing in foreign currency ($U_{A \rightarrow B}$) or issuing in home currency ($U_{A \rightarrow A}$), also as a function of $K$. The currency-switching costs $K_i$ are distributed over $[K, \infty)$ with a Pareto CDF $H(K)$. For illustration, the graph focuses on the case $\theta = 1$ and uses symmetric country fundamentals ($\lambda_j, G_j, F_j$). There are five equilibria.

and the dominant equilibria remain interior. Hence in this respect, our model is consistent with the evidence in Eichengreen et al. (2017) that despite currency dominance, multiple currencies co-exist as units of denomination in equilibrium.

3.3 Comparative Statics Analysis

We can now consider comparative statics with respect to country fundamentals. The following proposition characterizes these results.

**Proposition 3 (Comparative statics).** Consider equilibria of class BA (the results for class AB equilibria are symmetric). The following comparative statics results hold at an interior, stable equilibrium:

1. $G_A$: An increase in country A’s government bond supply $G_A$ increases foreign-currency issuance by B-firms ($\frac{\partial K}{\partial G_A} > 0$) for any $\frac{1}{2} < \theta \leq 1$. A sufficiently high increase in $G_A$ dissolves the no-entry equilibrium, resolving equilibrium multiplicity in favor of a high-entry equilibrium with currency A dominance.

2. $F_A$: The same holds for increases in the mass $F_A$ of firms in country A, such that $\frac{\partial K}{\partial F_A} > 0$, under the parameter assumption $(1-\theta) \left[ 1 + \frac{G_A}{F_A+F_B} \right] < \theta \leq 1$. This parameter assumption is more stringent than $\frac{1}{2} < \theta \leq 1$ and hence requires a sufficiently high degree of returns to scale.
3. \( \lambda_A \): The same holds for increases in the overall matching intensity \( \lambda_A \), so that \( \frac{\partial \hat{K}}{\partial \lambda_A} > 0 \) for any \( \frac{1}{2} < \theta \leq 1 \).

4. \( F_B \): The sign of the comparative static \( \frac{\partial \hat{K}}{\partial F_B} \) is ambiguous and depends on the level of entry. For sufficiently high entry \( \hat{K} \), the comparative static becomes unambiguously positive.

**Proof.** See Appendix Section A.3.

Figures 3a and 3b provide a graphical exposition of these comparative static results, again in the illustrative case with symmetric fundamentals and \( \theta = 1 \). Consider first an increase in government bond supply \( G_A \). As shown in Figure 3a, this acts as an outward shift of the blue curves \( \bar{U}_{B \rightarrow A} \) and \( \bar{U}_{A \rightarrow A} \): as government bond supply grows, the mass of investors \( m_{I,A} \) that are sellers in the A money markets increases, which in turn benefits issuers in currency A’s market as it expands the pool of liquidity available to them at date \( t_1 \). If this shift occurs starting from the high-entry class-BA equilibrium (point 2), the equilibrium threshold \( \hat{K} \) shifts further to the right, which means entry is increasing, and currency A’s dominance is more entrenched. If the shift occurs starting from the high-entry class-AB equilibrium (point 4), \( \hat{K} \) conversely shifts to the left, as the resulting asymmetry in fundamentals weakens currency B’s dominance.

Crucially, increasing \( G_A \) sufficiently will cause a qualitative shift in the configuration of equilibria in both equilibrium classes. First, a sufficient upward shift of the curve \( \bar{U}_{A \rightarrow A} \) will raise \( \bar{U}_{A \rightarrow A} \) above \( \bar{U}_{A \rightarrow B} \) over the entire domain \( [K, \infty) \) for class AB equilibria so that the two curves never intersect. Intuitively, this implies that sufficiently strong asymmetry between \( G_A \) and \( G_B \) dissolves all class AB equilibria in favor of class BA equilibria. Simply put, equilibria in which B is dominant cannot survive once country A achieves a sufficiently large advantage in government debt supply, all other fundamentals equal.

Second, large enough increases in \( G_A \) also impact the configuration of BA equilibria themselves. This happens once the \( \bar{U}_{B \rightarrow A} \) curve crosses above \( \bar{U}_{B \rightarrow B} \) at \( \hat{K} = K \). Once this threshold is crossed, the no-entry and low-entry equilibria (points 1 and 2) both disappear, leaving the high-entry equilibrium as the sole remaining one. These results illustrate another key economic point, which we discuss further below: multipolar equilibria (with denomination dispersed among multiple competitor currencies) only survive in a world of roughly symmetric fundamentals, while sharp asymmetries among countries result in dominance.

Next, consider an increase in the mass of firms \( F_A \) in country A, the effects of which are shown in Figure 3b. Growing the size of the private sector is *not* equivalent to increasing
Figure 3: Comparative statics

(a) Increasing government bond supply $G_A$

(b) Increasing firm mass $F_A$

Notes: We consider introducing asymmetry in country fundamentals by increasing government bond supply $G_A$ (Panel A) or the mass of firms $F_A$ (Panel B).
safe government debt supply: while these have the same effect from the perspective of class BA equilibria, the same is not true for class AB equilibria. Similar to an increase in $G_A$, increasing $F_A$ shifts up the curve $\bar{U}_{A\rightarrow A}$, since a share of the additional mass of A firms $(1 - H(\hat{K}))$ will continue to issue in home currency, improving the liquidity of the currency A markets. On the other hand, some of the additional mass will go towards improving the liquidity of currency B markets, in proportion to the share of A firms $(H(\hat{K}))$ that issues in foreign currency. For $\bar{K} > K$, increasing $F_A$ therefore tilts up the $\bar{U}_{A\rightarrow B}$ curve as well, which has the effect of increasing, rather than decreasing, entry of A firms into currency B.

The net impact of increasing $F_A$ on entry in class AB equilibria therefore depends on the relative strength of these two forces, which depends on the value of $\hat{K}$: for high value of $\hat{K}$, the second force will tend to be relatively stronger, and vice-versa. Figure 3b shows an example in which the starting value of $\hat{K}$ in the stable AB equilibrium is sufficiently high that the second force is stronger, and as a result, the increase in $F_A$ leads to overall higher entry, unlike in the case of increases in $G_A$.

These different forces illustrate a fundamental difference between sovereign and private issuance in our model: if a country does not start out as a dominant currency issuer, simply growing the size of the private sector is not guaranteed to facilitate the internationalization of its currency, and in fact can be counterproductive. Increasing the stock of safe government debt supply is instead a more reliable instrument, as it will facilitate international usage of the currency regardless of the starting equilibrium conditions.

3.4 Asset Pricing Channels and Dominance

In addition to matching the empirical evidence on quantities of debt issued in the dominant currency, our model also speaks to the relationship between asset prices, issuance, and dominance. The following proposition characterizes these asset pricing results.

**Proposition 4 (Asset pricing).** The convenience yields in the model satisfy the following properties in class BA equilibria (the results for class AB equilibria are symmetric):

1. Holding fixed country fundamentals, entry impacts convenience yields positively:

$$\frac{\partial(P_{0,A} - \beta^2)}{\partial \hat{K}} > 0.$$  \(29\)

A corollary is that, comparing across equilibria for a given set of parameters, equilibria with higher entry into currency A feature higher convenience yields in country A.
2. Increasing government debt supply $G_A$ impacts the convenience yield $P_{0,A} - \beta^2$ in two opposite ways: the resulting increase in liquidity demand ($m_{F,A}$) affects the convenience yields positively, while the resulting increase in liquidity supply ($m_{I,A}$) affects it negatively. Hence, depending on the relative strength of these two channels, the model can generate either a net negative impact of $G_A$ on the convenience yields or a net positive one.

Proof. See Appendix Section A.3.

The first part of Proposition 4 stresses that conditional on holding country fundamentals fixed, stronger dominance of the center country, as captured by entry into its currency, raises its convenience yield. In addition, comparing across equilibria, those equilibria which feature more entry will have higher convenience yields.

On the other hand, as shown in the second part of the proposition, increasing the stock of sovereign debt $G$, which alters fundamentals, can have ambiguous overall impact due to the presence of two opposing forces. Recall the expression for the convenience yield in country $A$, which is

$$P_{0,A} - \beta^2 = \frac{\lambda_A \beta(1 - \beta)}{2} m_{F,A}^\theta m_{I,A}^{\theta-1}.$$  \hfill (30)

When $\theta < 1$, both liquidity demand ($m_{F,A}$) and liquidity supply ($m_{I,A}$) impact the convenience yield: intuitively, rising liquidity demand increases the convenience yield, while rising liquidity supply reduces it. Figure 4a illustrates this point for a parametric case where $\theta = 0.9$: while entry is increasing in sovereign debt supply $G_A$, the convenience yield is decreasing in $G_A$. This relationship matches the empirical findings in Krishnamurthy and Vissing-Jorgensen (2012) that convenience yields are decreasing in government debt supplies.

The $\theta = 1$ case is a knife-edge parameter configuration in which the liquidity supply channel is shut off: in this special case, shifts in the liquidity supply schedule (such as changes in government debt supply $G_A$) do not have a direct effect on bond prices, but rather only impact them through their indirect effect on stronger equilibrium entry $\hat{K}$. Therefore, the convenience yield is unambiguously increasing in government debt supply $G_A$. Figure 4b illustrates this case where both the convenience yield (in blue) and entry, $\hat{K}_{\text{max}}$ (in orange), are increasing functions of $G_A$.

It is worth noting that the asset pricing channels of our model indicate that it is possible, if $G_A$ is sufficiently large relative to $G_B$, to have stable equilibria where $A$ has lower convenience yields, but it is nonetheless still dominant as a currency of denomination. In such a case, the issuance in currency $A$ is sustained not by the convenience yield force in
Notes: We focus on equilibria of the class BA, in which B firms switch to currency A. We show the behavior of the convenience yield $P_{0,A} - \beta^2$ for currency A as a function of government debt supply $G_A$. The analyses show these simulated comparative statics selecting the stable equilibrium with maximum entry threshold, $\hat{K}_{\text{max}}$, which is also plotted. Panel A uses $\theta = .9$, while Panel B uses $\theta = 1$. The rest of the parameter values in this example are $F_A, G_A, G_B, \lambda_A, \lambda_B = 1$; $\phi, K = .5$; $\alpha = 1.5$; and $F_B = .1$, where we set $F_B$ low to emphasize the impact of liquidity supply on convenience yields, minimizing the indirect entry effect.

In our model, but rather by the settlement benefit force, which is much higher in the dominant country. These two forces therefore allow our theory to accommodate the empirical finding that, according to certain measures in certain periods of time, US Treasuries may attain lower convenience yields than government bonds in other currencies (Diamond and Van Tassel, 2021).

### 3.5 Issuance and Complementarity in the Cross Section of Firms

In the baseline set of international equilibria outlined so far, firms trade off the costs of currency mismatch with both of the two benefits of dominant-currency issuance: favorable convenience yields and high settlement liquidity. In reality, issuers in the cross-section of firms may benefit disproportionately from one or the other incentive.

In this section, we present an extension of our baseline model that separates out these two roles according to the separate issuance incentives and formalizes the complementary nature of the denomination decisions of these two types of corporate borrowers. Hence we provide a positive explanation for why, today and as well as historically, we observe a wide range of different borrowers issuing debt in the dominant currency—from the safest global
borrowers all the way down to more speculative and risky ones. We also illustrate how these forces respond in heterogeneous ways to an increase in government debt issuance such that the strategic complementarities between these actors reinforce the dominant currency equilibrium.

We observe, for example, that global corporate borrowers with poor credit ratings issue debt in the dominant currency even though their debt is not sufficiently money-like to benefit from convenience yields. Nonetheless, these borrowers will still be attracted to issuing in the dominant currency because of its settlement liquidity. As such, these issuers act as net liquidity demanders in the dominant-currency money markets. On the other hand, particularly safe borrowers such as the German sovereign-backed issuer KfW may not have many settlement needs but still be drawn to dominant currency issuance primarily to capture the convenience yield, thereby acting as net liquidity suppliers.

Our analysis focuses on the class BA equilibria and separates the liquidity demand and liquidity provision roles in the cross-section of firms in country $B$. Specifically, we now allow the overall mass of firms $F_B$ to be composed of two different groups of firms:

- A first mass $F_B^+$ consists of pure liquidity suppliers: these are issuers for whom $\phi = 0$, which therefore never experience an early realization profits and hence have no motive for demanding liquidity and will not contribute to the liquidity-demanders masses $m_{F,j}$ in either country. The issuance incentive is purely a function of the convenience yield in each country:

$$u_{i,j}^{F_+} = \frac{\lambda_j \beta (1 - \beta)}{2} \theta m_{F,j} \theta m_{I,j}$$

(31)

- A second mass $F_B^-$ consists of pure liquidity demanders. These are firms whose bonds have no possibility of re-sale in the money market of date $t_1$, so that effectively $\lambda_j$ (which is now heterogenous for different assets) is zero for these firms’ issues. These firms therefore will not contribute to the liquidity-supplier masses $m_{I,j}$. The issuance incentive is purely a function of the settlement benefit in each country:

$$u_{i,j}^{F_-} = \frac{\lambda_j^+ \beta (1 - \beta)}{2} \sigma m_{F,j} \sigma m_{I,j}$$

(32)

The cost $K_i$ follows the same distribution $H(K)$ in these two subgroups of firms, and $F_B = F_B^+ + F_B^-$. These two groups of firms will now have two different endogenous equilibrium thresholds ($\hat{K}^+, \hat{K}^-$): liquidity suppliers issue in foreign currency if and only if $K_i < \hat{K}^+$, while liquidity demanders issue in foreign currency if and only if $K_i < \hat{K}^-$. 29
The equilibrium conditions pinning down the two thresholds of \((\hat{K}^+, \hat{K}^-)\) are:

\[
\lambda_A m_{F,A} \theta^{\theta-1}_F - \hat{K}^+ = \lambda_B m_{F,B} \theta^{\theta-1}_F,
\]

\[
\lambda_A m_{F,A} \theta^{\theta-1}_F - \hat{K}^- = \lambda_B m_{F,B} \theta^{\theta-1}_F, \tag{34}
\]

To complete the characterization of the equilibrium, the liquidity-demand masses in the two countries are

\[
m_{F,A} = \phi \left[ F_A + H(\hat{K}^-)F_B^- \right], \quad m_{F,B} = \phi \left[ 1 - H(\hat{K}^-) \right] F_B^- \tag{35},
\]

while the liquidity-supply masses are

\[
m_{I,A} = F_A + G_A + H(\hat{K}^+)F_B^+, \quad m_{I,B} = G_B + \left[ 1 - H(\hat{K}^+) \right] F_B^+. \tag{36}
\]

First, we illustrate the complementary nature of issuance by each type of firm. Consider the extreme case in which there is no mass of private liquidity suppliers \((F^-)\). If there is also no government debt \(G_j = 0\), the mass of liquidity supply \(m_{i,j}\) will be zero, and there will be no settlement benefits for liquidity demanders (equation 32). Since these firms do not benefit from the convenience yield, there is no incentive to issue debt, and \(F^- = 0\).

On the other extreme, consider the case where there is no mass of liquidity demanders \((F^-)\), meaning that \(m_{F,j} = 0\). Without demand for settlement, the mass of circulating medium \(m_{I,j}\) serves no purpose and therefore does not harvest a convenience yield. As a result, there is no utility to issuance by either type of firm and no private debt issuance. The issuance decisions of the two types of firms are therefore complementary in the cross-section, as formalized in the following proposition.

**Proposition 5 (Cross-sectional complementarities).** The equilibrium entry decisions of liquidity suppliers \((F_B^+)\) and liquidity demanders \((F_B^-)\) are complementary:

\[
\frac{\partial \hat{K}^+(\hat{K}^-)}{\partial \hat{K}^-} \geq 0, \quad \frac{\partial \hat{K}^-(\hat{K}^+)}{\partial \hat{K}^+} \geq 0, \tag{37}
\]

with the inequalities strict in an interior equilibrium (i.e., one which features positive entry by both types of firms). Hence strategic complementarity in the denomination decisions of these two classes of borrowers reinforces the dominant-currency equilibrium.

**Proof.** See Appendix Section A.3.

Second, we analyze the heterogeneous impact of increasing government debt on the
issuance incentives of these two types of firms. A positive supply of government debt \((G_j > 0)\) contributes to the liquidity supply \(m_{I,j}\) that circulates as a medium of exchange and is useful for settlement. The settlement benefit then raises the utility of debt issuance for the liquidity demanders, leading to the creation of liquidity demand \((F^-)\) in the same denomination. However, in the general case of \(\theta < 1\) where government issuance reduces convenience yields, private liquidity suppliers that only benefit from the convenience yield \((F^+)\) are crowded out (e.g., Greenwood et al., 2010; Roberds and Velde, 2016; Krishnamurthy and Vissing-Jorgensen, 2015). This crowding out force appears in a broad class of models in which the private sector incentive to create debt comes only from the convenience yield (e.g., Krishnamurthy and Vissing-Jorgensen, 2015; Gopinath and Stein, 2021; Gorton and Ordonez, 2022) and therefore only benefits firms that can create safe assets. In these papers, increasing the supply of dollar-denominated US government debt reduces private dollar debt issuance, thereby hampering the dominant equilibrium.

Our model also features an additional force where government issuance simultaneously raises the benefit of settlement such that private liquidity demanders are crowded in. As a result, the dominant equilibrium is sustained by the overall entry by both type of firms (Proposition 4), even though the entry by liquidity suppliers is partially offset by the impact on asset prices.

Figure 5 provides a parametric example illustrating the crowding-in and crowding-out effects. We plot the entry thresholds of liquidity demanders \((\hat{K}^-)\) and liquidity suppliers \((\hat{K}^+)\) as we vary \(G_A\) with \(\theta = .9\), keeping the other parameters the same as in Figure 4a. Entry by risky firms (the liquidity demanders) increases with \(G_A\), since these firms benefit by the improved liquidity of \(A\) markets but do not care about convenience yields. On the other hand, entry by safe firms (the liquidity suppliers) falls with \(G_A\), since these firms do not value the settlement liquidity benefits, but rather only care about the declining convenience yield.

4 Liquidity and Denomination Throughout History

In the last four centuries of the international monetary system, the dominant currency in global finance has been the Dutch florin followed by the British pound sterling and then the US dollar. We trace these transitions to large asymmetries in the supply of safe government-backed debt. We then show that the sovereign’s ability and incentive to invest in institutions that deepen financial market liquidity are larger in the center country that hosts the dominant
Figure 5: **Crowding in and crowding out in the model with firm heterogeneity**

\[ \dot{K}^+ \]
\[ \dot{K}^- \]
\[ G_A \]

**Notes:** We consider comparative statics for the model with additional firm heterogeneity. We plot the entry thresholds for liquidity suppliers \((\dot{K}^+)\) and liquidity demanders \((\dot{K}^-)\) as a function of government debt supply \(G_A\). The analyses show these simulated comparative statics selecting the equilibrium with highest entry. We use the same parameters in Figure 4a, with \(\theta = 0.9\). We set \(F_b^+ = F_b^- = 0.05\), so that the total mass of \(B\) firms is equal to the one used in Figure 4.

currency. These investments in turn generate additional complementarities that deepen the dominance regime. We then discuss the role of international trade throughout these regimes, which acts as an additional complementarity to dominance in finance. Finally, we consider welfare in the context of the Bretton Woods era liquidity coordination and the impact of modern policy tools such as central bank swap lines in the context of aggregate risk.

### 4.1 Transitions Throughout History

**Emergence of the First Global Currency.** Prior to the Dutch florin era in the 17th and 18th centuries, the Italian city-states of the Renaissance were the most similar to Amsterdam in being prominent in both trade and finance. Yet the Republics of Genoa, Venice, and Florence conducted their finance in a constellation of local currencies, such as the Genoese and Venetian lira and the Florentine florin, and none achieved the centrality that the Dutch florin would attain in subsequent centuries.

Our model rationalizes this difference between the Italian city-states and the later Dutch experience and attributes it to the economic mechanisms centered on increasing returns to scale in liquidity provision. The multipolar equilibrium of the several Italian currencies of roughly equal importance is stable as long as these currencies have separate and approxi-
approximately symmetric underlying liquidity pools (as determined by $G_j$). This system corresponds to equilibrium point 0 of Figure 2.

We consider this to be an accurate representation of the historical context in which the constellation of Italian currencies circulated in the form of physical coins. They were in fluctuating and uncertain supply in any given place and time because they were often transported, debased, or re-minted. Indeed, Italy during the Renaissance faced the same settlement frictions stemming from a large variety of metallic coins that Amsterdam faced later on. However, while these city-states had large banks (including the Bank of San Giorgio dating from 1407 in Genoa and the Medici Bank from 1397 in Florence), none of them invested in creating a large and steady supply of safe debt in a common currency ($G_j$) as Amsterdam did.\textsuperscript{13} The Genoese lira and the Florentine florin therefore remained unremarkable in terms of their liquidity properties like the other coins of the era.

In contrast, the Dutch florin was a pure unit of account that lived on the ledgers of the Bank of Amsterdam and therefore did not suffer any of the usage frictions of metallic coin. A florin was a safe claim issued by the Bank of Amsterdam and backed by the City of Amsterdam. All bills of exchange settled in Amsterdam were required to be denominated in florin, so the initial supply was sufficient to settle all of Amsterdam’s liquidity demand. In the context of our model, the financial innovations by the Bank of Amsterdam allowed it to increase florin-denominated $G$ sufficiently to trigger a shift from the multipolar equilibrium to unipolar dominance. It did so by pooling disparate assets (such as the various kinds of specie and coins that were used prior to the florin in trade) to back a single, unified money market in the florin unit of account. Moreover, the fact that the Dutch cities did not all compete with each others’ currencies, but rather followed Amsterdam’s lead with its florin, provided liquidity agglomeration that the Italian city-states lacked.\textsuperscript{14} In Section 4.3, we discuss Amsterdam’s incentives to invest in the florin technology in the first place.

The usefulness of the florin could also be measured in part by the \textit{agio}, the market exchange rate between locally circulating physical coin (current guilders) and bank florin. The \textit{agio} was steadily around 4.5 to 5\% for most of the 17th and 18th centuries (Van Dillen, 1934, p. 91, 102), a premium that in part captured the florin’s superior liquidity.

\textbf{Pound Sterling Dominance.} The Bank of Amsterdam weakened considerably after the Fourth Anglo-Dutch War and eventually collapsed following the French invasion of 1795,

\textsuperscript{13}Unlike Amsterdam, the Italian banks were slow to adopt payments by account transfers. For instance, it was not until 1675 that the Bank of San Giorgio in Genoa issued depositors transferable vouchers reflecting deposit accounts (Willis, 1943, p. 12).

\textsuperscript{14}See Appendix B.1 for more details on the arrangement between Rotterdam and Amsterdam, for example.
during which concerns about the florin’s backing were reflected in a drop in the agio to - 14%.\footnote{The Bank had secretly made large unsecured loans to the Dutch East India Company after the war, which had suffered large losses from the naval blockade in the war. When these loans and losses came to light, the Bank suffered a run from which it never recovered.} In our model, this is equivalent to a large and sudden contraction in $G_A$ that shifts in the $U_A$ curve and dissolves the dominant equilibrium of entry into $A$. A brief period followed without a clear dominant currency, but this was resolved by Great Britain’s victory in the Napoleonic Wars. The UK at that time had the largest economy and a government debt in pound sterlings of approximately 200% of GDP, which was coupled with a state capacity for taxation and credible governance that made this debt safe. As in the case of Amsterdam before it, having the largest pool of safe liquid assets for settlement initiated the process of dominance, which the UK held until the mid-20th century.

The closest likely competitor to British dominance was the French franc. However, France experienced several disruptions to its supply of safe government debt during the regime changes in the late 18th century, and its loss at Waterloo reduced its subsequent ability to float a large volume. Unlike the UK, it was required to pay off its wartime debt with much higher taxes instead of new long-term issuances, which were also at higher rates (\textit{Bordo and White, 1991}). While the franc itself returned to the gold standard such that the coins were just as safe as pounds sterling, the outstanding amounts of other forms of franc-denominated claims were much smaller than that of the pound’s. One illustration of pound dominance is that French firms were the largest foreign issuers in the corporate bond issuance in London, accounting for 25% on average from the mid-19th century to WWI.\footnote{Authors’ calculations based on bond data from the \textit{Investor’s Monthly Manual}.}

\textbf{Dollar Dominance.} Following WWI, \textit{Eichengreen and Flandreau (2009)} shows that the pound sterling and US dollar were held in roughly equal proportion by central banks as foreign exchange reserves throughout the interwar period. It notes that the dollar overtook the pound shortly after WWI (consistent with dollar liquidity growing following the centralization of US financial markets and a large war-time debt), but that its status also shrank after the 1933 dollar devaluation (consistent with a collapse in confidence in the US government’s commitment to its currency).

During the Bretton Woods era immediately after WWII, the dollar’s dominant status was a result of the institutional centrality of the dollar within the gold exchange standard. Afterwards, the dollar’s supremacy grew because the source of commitment shifted from the US gold reserves to its fiscal capacity, the latter of which was much larger. This increase in fiscally-backed commitment solidified the dollar’s dominance even further.
The emergence of the use of the euro in the late 20th century similarly reflects a consolidation of commitment, represented by $G$, across multiple countries into a single denomination. Similarly, the contraction of the euro’s global usage after 2008 matches a decrease in $G$, reflecting new information that government debt from countries like Spain, Italy, Portugal, and Greece did not contribute to the commitment backing the euro.

$F_j$ Throughout History. Throughout history, there have also been numerous examples of countries whose economic size alone was not sufficient to propel them to dominance, as conceptually illustrated in Proposition 3. For example, Spain was the largest and wealthiest global economy during the 17th century, and yet the much smaller City of Amsterdam emerged as the center of the international financial system. During the turn of the 20th century, the US overtook the British empire as the largest economy in the world (see Figure B.I in Appendix Section B.3). The projections now are that China will overtake US GDP in the next decade. Yet in all cases, within the language of our theory, these are examples of countries with large $F_j$ but small $G_j$. Spain lacked the government commitment and financial technology to create a liquid money market; the US financial sector and monetary system was fragmented until 1913; and China lacks both an internationally tradable base of government-backed money market instruments and confidence in its government’s commitment not to appropriate those assets.

4.2 Endogenous Investment in Financial Market Liquidity

Following the establishment of a dominant currency, the government of the dominant country has further incentives to innovate in institutions and financial technologies that advantage its market liquidity. These innovations are less costly to finance in the dominant currency, and they further entrench dominance. The history of the Bank of Amsterdam’s innovations in the florin, and the institutional changes in the London money market that impacted the pound sterling include several dimensions of these additional sources of complementarity.

We first specify the sovereigns’ objective functions. The government of country $j$ maximizes the following:\footnote{Note that investors always break even in equilibrium, so that we omit the utility of investors in writing the sovereign objective.}

$$ W_j = F_j \int u_i^F(K_i) \, dH(K_i) + G_j(P_{0,j} - \beta^2). \quad (38) $$
The first term in this objective function corresponds to the purely utilitarian welfare criterion that aggregates the preferences of domestic firms. In addition to this standard utilitarian objective, we allow the government to have a profit motive, which is reflected in the second term of equation (38). These seigniorage revenues correspond to the convenience yield on sovereign debt issued at \( t_0 \), which scales linearly with the size of government issuance \( G_j \). These revenues could be used to finance a costly investment in government-backed institutions such as ones that expand state or fiscal capacity.

For the rest of the analysis in the paper, we consider the case in which \( A \) is the higher liquidity market. We therefore focus on class \( BA \) equilibria—the only ones that remain present once asymmetry in fundamentals is sufficiently large. The analysis is of course symmetric for the case in which \( B \) is the higher-liquidity country. We now consider the sovereign’s incentives to expand its commitment to government-backed securities (increasing \( G_j \)).

**Proposition 6 (Sovereign incentives).** Consider an interior, stable equilibrium of the model of class \( BA \), with a higher convenience yield in the dominant country \( A \) than in country \( B \), and in which we hold seigniorage earned on existing (inframarginal) sovereign debt units fixed. There exists an entry threshold \( \hat{K}^\dagger \) such that if entry in the dominant currency is sufficiently high (\( \hat{K} > \hat{K}^\dagger \)), the sovereign’s incentives to invest in government commitment are larger in the dominant country:

\[
\frac{\partial W_A}{\partial G_A} > \frac{\partial W_B}{\partial G_B}.
\]  

(39)

The results for class \( AB \) equilibria are symmetric.

**Proof.** See Appendix Section A.3.

In analyzing these incentives, we assume that the sovereign receives the seigniorage revenue stream from the convenience yield at issuance. In this sense, the seigniorage aspect of the sovereign’s problem is akin to that of a durable goods monopolist (Coase 1972): the existing units of government debt are inframarginal and hence variation in the convenience yield on old issuance does not affect the revenues on them, but the choice of \( G_j \) does affect the seigniorage revenues earned on newly issued debt.\(^{18}\)

\(^{18}\)If we were to consider the case in which inframarginal debt is also re-priced (as for a sovereign with commitment and long planning horizons), there would be an additional tradeoff, as increasing \( G \) would expand the revenue base, while at the same time reducing the per-unit revenues earned on all units of debt, as in a Laffer curve. The strength of this mechanism would depend on the elasticity of demand for the sovereign bonds.
The preceding proposition demonstrates that the government in the dominant country has a greater incentive to undertake costly investments in commitment technologies that allow it to expand $G$. For example, investing in the military strengthens the government’s access to resources, thereby expanding fiscal capacity and the ability to issue safe assets. This occurs because of two forces. First, the sovereign in the dominant country faces a higher marginal value from additional debt issuance, in terms of seigniorage revenues, because of the higher convenience yield. Second, the non-dominant country has a lower incentive because its liquidity expansion does not benefit the set of inframarginal domestic firms that have already chosen to issue in the dominant currency. An additional source of complementarity, in practice, is that the government’s ability to invest is also higher in the dominant country, because the convenience yields that the government earns generates a larger revenue stream that can reinforce the government commitment (Jiang et al., 2020b).

Historically during the period of pound dominance, the British government issued debt at the most advantageous terms of any sovereign, which gave it a funding advantage for military and naval growth. These investments enabled further imperial expansion that increased the government’s tax base and access to resources, thereby expanding its fiscal capacity to issue more debt. Additional debt issuance then deepened the asymmetry of Britain’s financial market depth relative to other countries’, drawing in more firms into the pound equilibrium and lowering the costs of financing future investment for all domestic firms. This positive dynamic feedback loop leads to the endogenous persistence of dominant regimes.

As we discuss in the next section, financial development has taken the form of increases in government safe asset supply as well as improvements in the capacity of the private sector to issue safe assets. In Appendix Section A.5, we also consider an extension of the model in which we allow for innovations that increase the pledgeability of private revenues. As in the case with the government’s commitment technology, we show that the incentive to invest in firms’ revenue pledgeability is larger for the dominant country.

Equipped with the theoretical result above, we now discuss several ways in which this interaction between currency dominance and incentives to invest and financially innovate have appeared throughout history.

4.2.1 Investments in Commitment Technologies Throughout History

Innovations and Complementarities in Amsterdam. The Bank’s introduction of a specie-florin repo facility in 1683 is credited with propelling the florin to global prominence (Quinn and Roberds, 2014a). Before this facility, obtaining florin required depositing specific
coins, and accounts were primarily held by those with liquidity needs in florin-denominated bills of exchange.

The specie repo facility was an investment in financial technology that greatly expanded access to florins by allowing individuals and businesses to monetize safe but illiquid assets. In the context of the model, the trust that the collateral posted in the repo facility would not be appropriated by the Bank of Amsterdam was key to its success, thereby generating $G_A$ and deepening the florin equilibrium.

The popularity of this facility, which doubled the quantity of florins at the Bank of Amsterdam despite the fees charged to use it, also illustrates the complementarity in the issuance incentives that we model. For specie investors, issuing florin against their assets was profitable because of the convenience yield, which captures the liquidity benefits of transacting in florin, reflecting the incentives to provide liquidity even by those without payment needs.

**Bank of England’s Changing Role.** In the UK, the evolving role of the Bank of England from its creation in 1693 to the 19th century encapsulates the institution’s changing incentives to facilitate financial market liquidity. At its founding, it was a private corporation that was granted several privileges in return for raising and administering the Crown’s debt, and during the early part of its history, the Bank competed with other private banks such that it sometimes limited market liquidity in order to protect its own balance sheet.

Beginning in the 1830s, a series of legislative reforms changed the Bank of England’s role into one of a liquidity supplier. First, the Bank’s notes became legal tender for pound sterling debts in 1833, which legally expanded the supply of pounds sterling. Second, the entire note issuance was consolidated onto the Bank’s balance sheet and given a large fiduciary issue in 1844, which again expanded the supply of pound-denominated settlement instruments. Third, by the mid-19th century, the Bank of England established its role as

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19 The quantity of florin doubled from approximately eight million to sixteen million from the mid 17th to the beginning of the 18th century and is credited with drawing Europe’s specie trade to Amsterdam, where it could be more profitably be conducted.

20 The privileges restricted banking competition and gave the Bank of England a monopoly over note issuance. From 1697 until 1844, only the Bank could raise equity; all other banks were restricted to partnerships of six or fewer (after 1844, this was altered to a radius of 25 miles around London). In 1708, the Bank was granted an exemption to laws restricting bank note issuances to private partnerships (Broz and Grossman, 2004). The Bank’s abuse of its monopoly during the 1825 crisis led to the Banking Act of 1826, which mandated Treasury monitoring of small-denomination note issuance.

21 Like most currencies during this era, the pound sterling referred to a specific metallic coin, and obligations denominated in sterling were contracted to be repaid in those coins. However, coins were inconvenient for the reasons already discussed, and private banks like the Bank of England found it profitable to issue paper notes denominated in sterling (i.e., claims on sterling coin).
a reliable lender of last resort to the financial sector (an innovation that Bagehot (1873) credits to the Bank), where its balance sheet explicitly became the backstop to the private bills market.

**Innovations and Complementarities in the London Money Market.** Institutionally, the private bills market also underwent changes. The legal codification of the contractual terms for bills of exchange coordinated the market on the terms of borrowing and the procedures for default, which reduced their information sensitivity (Dang et al., 2017) and collectively raised the safety and liquidity of the London money market. The growing clarity on the Bank of England’s discount window rules also helped to homogenize money market securities and raised the incentives to produce high quality “discountable” bills.

The government lowered the costs for the banking sector to create bills of exchange by deregulating private banks’ equity issuance in 1830. This fueled British banks’ overseas expansion, which took advantage of the growth in world trade during this period. London banks pursued a business model of issuing bills of exchange collateralized on the large base of international trade, which simultaneously allowed them to capture the dual benefits of issuance: the convenience yield for creating money-like assets in London and the settlement benefits from providing credit to exporters around the world (Xu, 2022). Their success relative to competitors from other nations was due to their access to the London bills market where they re-discounted the bills they underwrote. As in Amsterdam, these private and government investments in commitment and pledgeability monetized a pool of previously illiquid assets, further increasing market depth in the London money market.

**Innovations and Complementarities in the Dollar Market.** Innovation in the US financial system has allowed privately-issued short-term debt instruments to add to the pool of dollar-denominated money market liquidity. The growth of the US banking system and of the commercial paper market (Greenwood and Scharfstein 2013) are early examples of these liquidity-producing financial technologies. In more recent decades, securitization (Mian and Sufi 2009, Keys et al. 2010) allowed for further private-sector production of safe liquid assets, while the expansion of repo markets (Gorton and Metrick 2012, Krishnamurthy et al. 2014) broadened liquidity in the overnight segment of dollar money markets. Our study demonstrates that the financial and central banking advancements in the United States are a natural consequence of the incentives that come with being the issuer of a dominant currency, and that these developments further reinforce the currency’s dominance.
4.3 Finance and Trade

It has been clear both historically and today that trade and finance are intricately connected. Historically, bills of exchange were issued first and foremost to finance international trade transactions and today, trade finance continues to be an important part of the global credit market. This section clarifies how trade intensity interacts with the benefits from financial market depth, and how trade invoicing impacts the forces we identify in our model. We then discuss the history of trade and finance through the lens of our model.

4.3.1 Trade Invoicing in the Model

In our model, firms only receive revenues in their domestic currency. However, we can consider the case where firms receive a portion of their revenue stream denominated in foreign currency, as in the case of traded goods that are invoiced in dollars. For a given firm, this will reduce $K_i$ because the receipt of some profits in the foreign currency reduces the amount of currency mismatch that a firm faces if it issues dollar-denominated debt, as in our Appendix Section A.1 modeling of $K_i$.

A reduction in $K_i$ for all $i$ is a leftward shift of $H(K)$, raising the utility of issuing abroad. Rewriting the equilibrium condition in equation (25) in terms of firm masses, considering the $\theta = 1$ case for ease of illustration, the equilibrium condition is:

$$\lambda_A \phi \left[ 2F_A + G_A + 2F_B H(\hat{K}) \right] - \hat{K} = \lambda_B \phi \left[ G_B + 2F_B (1 - H(\hat{K})) \right] \quad (40)$$

Consider the case where there is an increase in the number of $B$ firms receiving revenues in foreign currency. The cost distribution shifts left from $H_0(K)$ to $H_1(K)$ where $H_0$ f.o.s.d. $H_1$. This shift increases $H(\hat{K})$ and raises $U_{B\rightarrow A}$ while reducing $U_{B\rightarrow B}$ for the marginal firm at $\hat{K} = \hat{K}_0$. This benefit of entry will lead $B$ firms to issue abroad until a new equilibrium is reached that equates the two sides, which will occur at $\hat{K}_1 > \hat{K}_0$. Therefore invoicing in foreign currency leads to more financial dominance.

Separately, we can also consider how the denomination of trade invoicing might endogenously emerge in our model as a consequence of the liquidity forces discussed.\textsuperscript{23} Although this steps outside our formal analysis, if firms were also allowed to choose their currency of

\textsuperscript{22}This argument generalizes straightforwardly to the general $\theta$ case, in which the results are analogous.
\textsuperscript{23}Goldberg and Tille (2008) and Gopinath et al. (2010, 2020) document the dominant role of the dollar in invoicing trade. Additional theoretical models of dollar use in trade invoicing include Rey (2001), Amiti et al. (2022), and Mukhin (2022).
invoicing, dominant currency invoicing would emerge because of the foreign exchange exposure firms have in their financial obligations. In that case, firms would choose to invoice in the foreign currency in order to reduce $K_i$, and the invoicing dominance is a byproduct of our equilibrium with financial dominance.

We note that our theory implies particular prescriptions for the paths that lead a currency to become globally dominant. For example, the theory suggests that in order to internationalize the renminbi, the Chinese government will need to open the capital account and invest in liquidity and financial innovation. In contrast, promoting trade invoicing in the renminbi would only spur debt market adoption to the extent that it reduces the cost of renminbi-denominated consumption.

4.3.2 Finance and Trade Through History

The preceding argument is that trade invoicing is a natural consequence of the liquidity forces of our model. While in the post-WWII dollar dominant world, the US has been both the largest country in international trade and had the deepest financial markets, history provides examples where volumes of trade invoicing was unlikely to have been the seed of currency dominance.

In the 17th and 18th centuries, the value of Spanish trade was a factor of two to six times larger than that of Dutch trade, and in fact, Dutch trade remained smaller throughout its centuries of florin dominance.\(^{24}\) The second global currency, the pound sterling, gained dominance in the 1820s during the first sovereign debt boom shortly after Waterloo, before the mid-century surge in world trade and expansion of the British Empire.\(^{25}\) Historically, world trade was also a much smaller component of world GDP: it was approximately 5% until the 1880s, at which point it grew to 10%.\(^{26}\)

In both the Amsterdam and British cases, there were investments in financial technologies by the state and private sector that enhanced liquidity. As we have explained, the Bank of Amsterdam’s ledger technology created liquidity in florin, which was the driving factor behind florin dominance in this period. For Britain, we described the legal and institutional developments around bills of exchange and the Bank of England that allowed international

\(^{24}\)In 1700, the ratio of Spanish to Dutch trade was just over two; by 1750 it had grown to almost six, and in 1800 it was still approximately five. See Appendix B.1.1 for the full sources and calculation.

\(^{25}\)As of 1815, the British Empire was still relatively small with little presence in Africa, the middle east, and Asia outside of India. Its land mass and population as a share of the world did not reach their peaks until the early 20th century with most gains in the second half of the 19th century.

\(^{26}\)Following the interwar years, world trade did not regain this share of GDP until 1970, and it is now at approximately all-time highs of 25%.
Figure 6: Values of cross-border financial claims in USD and total world exports

Notes: This figure plots the total values of cross-border debt securities denominated in US dollars with the total values of world exports at a quarterly level, and the ratio of the two series.

banks to securitize trade flows into forms of safe, liquid, money market instruments. During this period, British liquidity denominated in pounds underwrote 90% of world trade, while Britain in the goods market only comprised 50% of world trade (Xu, 2022).

In today’s dollar world, the volume of cross-border financial assets trade dwarfs the volume of real goods trade. Figure 6 plots the evolution of cross-border bank liabilities denominated in US dollars from 1975 until today relative to total world exports at a quarterly level. The average ratio in the last twenty years has been two. Accounting for the fraction of world trade that is denominated in US dollars (approximately 40% in recent years), the relative sizes are five-fold. Considering all debt securities in the world outstanding, the ratio of dollar-denominated financial claims to world trade annually has on average been eight-fold larger since 2015.

The relative volumes of financial assets and international trade in all currencies are similarly skewed. Annual global trade flows are approximately 24 trillion USD compared to global debt contracts outstanding of approximately 300 trillion USD. Assuming an average maturity of seven years and an average interest rate of 7%, annual debt payments are approximately 65 trillion USD, or three times global trade flows.
4.4 Liquidity Coordination and Welfare

At various points in history, there has been the recognition that one country may not be able to sustain the creation of enough liquid assets to support the pace of real economic growth. Triffin (1978) provided a prominent discussion of this concern in the context of gold-backed dollar reserves in the Bretton Woods period. Our model allows us to discuss liquidity provision policy and welfare from the perspective of a global planner. We continue to consider the case in which $A$ is the higher liquidity market, focusing on class $BA$ equilibria and on the case where $\theta = 1$. We consider a planner whose objective is utilitarian over the preferences of the two governments:

$$W = W_A + W_B. \quad (41)$$

Our first result is that the planner’s choice of optimal entry features more entry into currency $A$ than the competitive equilibrium.

**Proposition 7 (Global welfare).** Consider the case $\theta = 1$. Let $K^*$ be the value of $\bar{K} \in [K, \infty)$ that maximizes global welfare $W$, and let $\hat{K}_{\text{max}}$ be the stable interior equilibrium point featuring highest entry into the dominant currency.\(^{27}\) It holds that

$$K^* > \hat{K}_{\text{max}}. \quad (42)$$

**Proof.** See Appendix Section A.3.

At the heart of this result is the liquidity externality discussed in Section 2: entry into currency $A$ by firms in country $B$ carries social benefits in terms of improved market thickness, which are in excess of the private benefits of entry. Since these excess liquidity benefits are not internalized by the firms, there is too little private entry in equilibrium.

Conceptually, our setting presents an analogy with the theory of natural monopoly (Posner 1978). Here the higher-liquidity country ($A$) has aspects of a natural monopolist, since consolidating issuance in its currency is welfare-improving from a global perspective. An important difference between our model and theories of natural monopoly, however, is that in this setting first-best equilibria are also interior, as in Eichengreen et al. (2017). The planner’s chosen equilibrium $K^*$ achieves the first best, which can always be made into a Pareto improvement relative to the private equilibrium $\hat{K}_{\text{max}}$ by introducing appropriate

\(^{27}\)In the illustration in Figure 2, this corresponds to equilibrium point 2 when considering class $BA$ equilibria.
Similarly to the normative results that feature in theories of natural monopoly, optimal policy in this setting will therefore feature a subsidy to entry into currency $A$.

In the results above we have considered the entry problem from the perspective of the global planner. A second, related question is how the shadow value of increasing government debt supply $G_A$ in the leader country differs when viewed from the perspective of the global planner versus the sovereign in country $A$. That is, we are interested in comparing the two quantities $\frac{\partial W}{\partial G_A}$ and $\frac{\partial W}{\partial G_A}$. If the shadow value from the global planner’s perspective, $\frac{\partial W}{\partial G_A}$, is higher, then the planner will prefer to increase $G_A$ even beyond what is privately optimal for country $A$’s government, leaving open the possibility of welfare-improving international coordination in sovereign liquidity provision.

Proposition 8 (Liquidity coordination). Consider the case $\theta = 1$. In a stable, interior, class $BA$ equilibrium, the shadow value of increasing $G_A$ is higher from the global planner’s perspective, as compared to the perspective of the sovereign in the leading country ($A$), if and only if the following is satisfied:

$$H(\hat{K}) \frac{\lambda_A}{\lambda_B} > \frac{1}{2} \frac{G_B}{F_B} + \left[1 - H(\hat{K})\right] \iff \frac{\partial W}{\partial G_A} - \frac{\partial W}{\partial G_A} = \frac{\partial W}{\partial G_A} > 0.$$  \hspace{1cm} (43)

The results for class $AB$ equilibria are symmetric.

Proof. Differentiate $W_A$, $W_B$ with respect to $G_A$, and rewrite to yield the inequality. \hfill \square

In the expression above, note that if $A$ is dominant, then $H(\hat{K})$ is high, tending to one, while $1 - H(\hat{K})$ tends to zero, and likewise the ratio $\frac{\lambda_A}{\lambda_B}$ is high, increasing the likelihood that this condition is satisfied. The direction of this result therefore hinges on the relative magnitudes of $G_B$ and $F_B$. Improving liquidity in country $A$ has two effects on welfare in country $B$: on the one hand, it improves the utility of the infra-marginal $B$ firms that have already switched to foreign currency, but on the other hand it reduces the convenience yields earned on sovereign issuance $G_B$ by inducing stronger entry. If country $B$ has large private borrowing needs relative to the stock of safe government debt outstanding, the first effect outweighs the second, so that increasing $G_A$ is also welfare-improving from the perspective of country $B$.

If the condition in equation (43) is satisfied, the global planner will want to engineer incentives for country $A$ to further increase its liquidity supply, financing these with transfers from country $B$. These results provide a lens to interpret historical international liquidity transfers.\textsuperscript{28} We note that utility is transferable in our setting owing to the quasi-linear structure of preferences.
provision arrangements, such as the Bretton Woods agreements of 1944. In the Bretton Woods system, major world economies effectively coordinated on having liquidity provided by the United States with the bulk of the gold reserves underpinning the Bretton Woods gold standard held at the Federal Reserve.

Throughout the Bretton Woods period, the United States held more than 90 percent of the world’s gold reserves (Monnet and Puy 2020), with large transfers of gold from central banks outside of the United States to New York during and after WWII. These transfers and the resulting coordination on a US-backed gold convertibility system provide a historical counterpart to the possibility of welfare-improving coordination in international liquidity provision that features in our model. In response to the classic Triffin (1978) dilemma that the US gold reserves would be insufficient to back its internationally held liabilities, our model would have prescribed more liquidity coordination in the form of transfers of commitment (i.e., gold) to the United States.

4.5 Aggregate Risk, State-Contingent Liquidity, and Default Risk

Our discussion of liquidity provision did not draw a distinction between state-contingent and non-contingent expansions of liquidity supply $G_A$, although state contingency also played an important role within the Bretton Woods architecture—for instance, through the role of central bank swap line arrangements, which remain a core feature of the international monetary system to this day. A formal discussion of this topic requires extending the model to incorporate a role for aggregate risk, and hence we now suppose that $\phi$ is subject to an aggregate shock, realized at time $t_1$. The state is $\omega \in \Omega$, and in state $\omega$, which has probability $q_\omega$, the early profit realization probability $\phi$ takes on the value $\phi_\omega$. The state realization is a shock to aggregate liquidity demand: if the realized value of $\phi$ is higher, more firms experience timing mismatch and therefore there is more overall demand for liquidity.

We analyze this extended version of the model in the $\theta = 1$ case. We allow for state-contingency in the supply of government assets $G_A^A$ in the leading country, but we assume that $G_B, F_A, and F_B$ are all not state-contingent. The value of $G_A$ in each state $\omega$ is $G_A^\omega$. The following result presents the equilibrium indifference condition in this extended version of the model.

**Proposition 9 (Aggregate risk).** Consider the case $\theta = 1$. In the model with aggregate risk, focusing on the case in which $A$ is the dominant currency (class $BA$ equilibria), the
equilibrium condition that determines the marginal firm $i$ with $K_i = \hat{K}$ is

$$
\lambda_A \left( \mathbb{E}[\phi_\omega] (2(F_A + H(\hat{K})F_B) + \mathbb{E}[G^A_\omega]) + \text{Cov}[\phi_\omega, G^A_\omega] \right) - \hat{K} = \lambda_B \mathbb{E}[\phi_\omega] \left( 2(1 - H(\hat{K}))F_B + G_B \right).
$$

(44)

The results for class AB equilibria are symmetric.

Proof. See Appendix Section A.4.

With the addition of aggregate risk, the equilibrium condition is nearly the same as in the baseline case, except for the additional terms $\mathbb{E}[G^A_\omega]$ and $\text{Cov}[\phi_\omega, G^A_\omega]$, which are respectively the expected value of $G^A$ and its covariance with the timing mismatch probability $\phi_\omega$ across states. The intuition for the expectations term is straightforward: higher average government debt supply $G^A_\omega$ shifts the left-hand side of this equality upwards and thereby increases the equilibrium entry threshold $\hat{K}$.

The covariance term is positive when liquidity supply $G^A_\omega$ increases in states of the world with high liquidity demand (and hence high $\phi_\omega$). When the covariance is zero, even with stochastic $\phi_\omega$, the model collapses to the baseline case with constant $\phi$. When $G^A_\omega$ is positively correlated with $\phi_\omega$, the extra covariance benefit makes currency $A$ more attractive.

Central bank institutions such as the discount window and swap lines arrangements achieve positive covariance: both technologies allow the dominant currency issuer to expand liquidity supply when liquidity demand is particularly high.\(^{29}\) Swap lines provide emergency liquidity supply by one central bank to others: overwhelmingly, the central bank on the supplying end of these arrangements has been the US Federal Reserve, which provided US dollar liquidity in overseas markets during stress periods such as the global financial crisis of 2008-09 and the COVID crisis of 2020. These state-contingent expansions of dollar-denominated money market liquidity increases the expected liquidity benefits from issuing dollar debt and reinforces the dollar dominance equilibrium. As a result, they can be an important part of the architecture of the international monetary system (Bahaj and Reis, 2021).

The analysis also allows us to discuss default risk in government debt and the importance of the safety of the settlement asset. From the standpoint of $t_0$, default risk is that $G^A_\omega$ can fall in some states of the world. One effect of this risk is to reduce $\mathbb{E}[G^A_\omega]$, thereby reducing government supply and entry $\hat{K}$. A second effect is that if risk is higher in high-

\(^{29}\)It is worth noting that the innovation of a central bank acting as a lender of last resort occurred in London during the period of British pound dominance while the latter has been an innovation of the Federal Reserve system during dollar dominance.
liquidity states of the world, then the covariance term is negative, further reducing effective
government supply and $\hat{K}$. Hence higher perceptions of sovereign default risk hinder a
currency’s internationalization.

These effects concern risk realized between $t_0$ and $t_1$. There is a third effect that concerns
risk realized between $t_1$ and $t_2$ that does not arise in the modeling of this section, but is likely
an important concern. Bonds purchased at $t_1$ as settlement instrument for private debt at
$t_2$ are poor settlement instruments if these bonds carry default risk. For example, if between
$t_1$ and $t_2$, the bond defaults completely, then the firm will not own an asset to extinguish its
debt. At $t_0$, default risk in government debt will mean that the currency is a less attractive
currency in which to denominate private debts.

5 Conclusion

Our theory of the liquidity force that generates and sustains currency dominance rationalizes
the historical emergence and persistence of regimes over the last four centuries as well as
the transitions between them. The key feature that distinguishes the Dutch florin, the
British pound, and the US dollar during their respective periods of dominance is the depth
of the financial markets in those currencies. Whether this depth was intentionally created
through investing in financial technologies, as in the Dutch case, or it was the outcome
of global disruptions in geopolitical power as in the British and American cases, a large
homogeneous pool of safe government-backed securities has been the springboard for each
dominant currency. Once dominance is established, the ability and incentives to invest in
institutions that facilitate financial market depth are greater in the center country and lead
to greater entrenchment of the dominant regime.

In the second half of the twentieth century, the US dollar was established as the central
currency of the international monetary system, and today, the dollar remains the de facto
center. Looking into the future, our model has two implications for a modern Triffin dilemma
in which world growth may be outpaced by the ability of the US government to create safe
dollar liabilities. First, as in Despres, Kindleberger and Salant (1966), financial institutions
that speed up settlement, such as well-functioning repo markets, reduce the burden on
government debt as the primary source of liquidity. Second, private firms, both domestic
and foreign, contribute to the stock of dollar-denominated money-like liabilities. However,
we recognize that the latter does so by accumulating currency mismatch risk on balance
sheets around the world, which can have costly financial spillovers, as discussed in Jiang et
Therefore while US GDP may not be the limiting factor, the world continues to face barriers in costlessly creating safe liabilities in a foreign denomination.

An often-asked question in academic and policy discussions is whether the Chinese renminbi might be poised to displace the US dollar as the world’s dominant international currency in the near future (Horn et al. 2021, Clayton et al. 2022). Our theory suggests that China’s safe debt markets are currently not sufficiently liquid, safe, or investable in size to challenge the dollar’s status as a reserve currency.

References


\[^30\] On the topic of over-borrowing externalities in an open economy context, see also Caballero and Krishnamurthy (2001), Lorenzoni (2008), Bianchi (2011), and Korinek (2018).


Farhi, Emmanuel, Pierre-Olivier Gourinchas, and Hélène Rey, *Reforming the international monetary system*, CEPR, 2011.


A.1 Microfoundation for Switching Cost $K_i$

Assume that at date $t_1$, the foreign exchange rate $E$ either appreciates or depreciates with equal probability. Thus, a firm in country $B$ can convert its revenue to currency $A$ at a stochastic FX rate $E \in \{1 + \gamma, 1 - \gamma\}$. Suppose that a firm in $B$ chooses to issue debt in $A$. At $t_1$, the exchange rate realizes, with a depreciation of currency $B$ to $1 - \gamma$ being a bad state for the firm: the firm has revenues in units of currency $A$ of $1 - \gamma$ and debt obligation of one. We assume that in this bad state, the firm-i can pay a disutility cost of $\kappa_i \gamma > 0$ to make up for the lost revenue. The disutility cost is a modeling device that ensures that firms reckon some cost due to currency mismatch, and to ensure that the firm does not default so that the bond is riskless (and hence private bonds are perfect substitutes for government-issued bonds). We assume that there is heterogeneity in this cost across firms. In this case,

$$K_i = \frac{1}{2} \gamma \kappa_i$$  \hspace{1cm} (A.1)

which readily maps back to the model in the main text.

A.2 Conditions for Equilibrium Existence and Convexity

This section provides a proof for the equilibrium characterization given in Proposition 2. We discuss the conditions required for equilibrium existence and stability, as well as the second-order conditions associated with the optimization problems that feature in the model. We show that under the Pareto form for the distribution $H(K_i)$ featured in our baseline parametric analyses, the model’s objective function is well-behaved.

Consider equilibria of class $BA$ (as the analysis is symmetric for $AB$ equilibria). The marginal firm with $K_i = \hat{K}$ satisfies

$$\lambda_A (m_{F,A}m_{I,A})^{\theta - 1} [m_{F,A} + \phi m_{I,A}] - \hat{K} = \lambda_B (m_{F,B}m_{I,B})^{\theta - 1} [m_{F,B} + \phi m_{I,B}],$$

while the market clearing conditions are

$$m_{F,A} = \phi \left[ F_A + H(\hat{K})F_B \right], \quad m_{F,B} = \phi \left[ 1 - H(\hat{K}) \right] F_B,$$ \hspace{1cm} (A.3)

and

$$m_{I,A} = G_A + F_A + H(\hat{K})F_B, \quad m_{I,B} = G_B + \left[ 1 - H(\hat{K}) \right] F_B.$$ \hspace{1cm} (A.4)
Define

\[
\Delta(\hat{K}) = \lambda_A (m_{F,A} m_{I,A})^{\theta-1} [m_{F,A} + \phi m_{I,A}] - \lambda_B (m_{F,B} m_{I,B})^{\theta-1} [m_{F,B} + \phi m_{I,B}] - \hat{K}. \tag{A.5}
\]

The equilibrium condition is then that

\[
\Delta(\hat{K}) = 0. \tag{A.6}
\]

Since \( \Delta > 0 \) implies a positive entry incentive, a stable equilibrium occurs when \( \Delta(\hat{K}) \) has a root which crosses zero from above (so that \( \Delta' < 0 \) at the root). Note that the function \( \Delta(\hat{K}) \) becomes negative for large \( \hat{K} \):

\[
\lim_{\hat{K} \to \infty} \Delta(\hat{K}) < 0. \tag{A.7}
\]

Now consider two cases:

1. If the parameters \( \Theta \) are such that \( \Delta(\hat{K}) > 0 \), then the existence of at least one interior stable equilibrium is guaranteed by the intermediate value theorem, given that \( \Delta(\hat{K}) \) is continuous.

2. If the parameters \( \Theta \) are such that \( \Delta(\hat{K}) \leq 0 \), then a corner equilibrium occurs at \( \hat{K} = K \), which is stable if the inequality holds strictly (if \( \Delta(\hat{K}) < 0 \)) and unstable otherwise (if \( \Delta(\hat{K}) = 0 \)). Additionally, at least one interior stable equilibrium exists if and only if \( \Delta(\hat{K}) \) attains a positive value for at least some \( \hat{K} \), again by the intermediate value theorem.

In the second case, in which \( \Delta(\hat{K}) \leq 0 \), the existence of an interior stable equilibrium then hinges on whether \( \Delta(\hat{K}) \) attains a positive value at some point over its domain \((\hat{K}, \infty)\). A necessary condition is that \( H(\cdot) \) is sufficiently curved for low values of \( \hat{K} \).

To see that the Pareto distribution, which we use in our baseline parametric analyses, can satisfy the necessary curvature condition (in a parameter-dependent way), it suffices to give an example. Figure A.1 shows such an example where the Pareto form attains a positive value for \( \Delta(\hat{K}) \) given \( \Delta(\hat{K}) < 0 \). The example uses symmetric country fundamentals: it is then clear that an interior, stable class \( AB \) equilibrium exists as well, in addition to the stable class \( BA \) equilibrium. This demonstrates that stable equilibria of all three classes outlined in Proposition 2 (interior class \( BA \), interior class \( AB \), and no-entry) can exist simultaneously for a given parameter vector \( \Theta \), as desired.\(^1\)

To gain an understanding on the conditions on \( H(\cdot) \), it is also helpful to study the \( \theta = 1 \) case, in which it is possible to provide an analytical characterization. When \( \theta = 1 \), we obtain that:

\[
\Delta(\hat{K}) = \phi \lambda_A [2F_A + G_A] - \phi \lambda_B [2F_B + G_B] + 2\phi F_B (\lambda_A + \lambda_B) H(\hat{K}) - \hat{K}. \tag{A.8}
\]

Under strictly stronger \( A \) fundamentals \((F_A > F_B, G_A > G_B, \lambda_A > \lambda_B)\), the term marked (1) is positive and constant. The term marked (2) is zero at \( \hat{K} = K \), and it asymptotes to \( 2\phi F_B (\lambda_A + \lambda_B \rightarrow \infty) \). The properties in this example can also be obtained with asymmetric country fundamentals: for instance, small perturbations of the parameters in Figure A.1 will continue to yield double crossings of the horizontal axis.

---

\(^1\) The
Figure A.I: Existence and stability: numerical example

Notes: This figure plots the function $\Delta(K)$ taking $H(\cdot)$ as a Pareto distribution, and for the following choice of parameters with symmetric country fundamentals: $F_A = F_B = G_A = G_B = \lambda_A = \lambda_B = 1$, $\theta = .95$, $K = .5$, $\phi = .5$, $\alpha = 1.5$. The black dots correspond to stable equilibria (no-entry and interior of class $BA$). By symmetry, an interior class $AB$ equilibrium also exists.

$\lambda_B > 0$ as $\hat{K}$ goes to infinity. The second term is also strictly increasing.

A necessary condition on $H(\cdot)$ for $\Delta(\hat{K})$ to attain a positive value (assuming the case $\Delta(K) < 0$) is then that

$$\frac{d}{d\hat{K}} \left( 2\phi F_B(\lambda_A + \lambda_B)H(\hat{K}) \right) > 1,$$

or

$$2\phi F_B(\lambda_A + \lambda_B)h(\hat{K}) > 1$$

for low values of $\hat{K}$. Candidate distributions are ones for which

$$h(K) > \frac{1}{2\phi F_B(\lambda_A + \lambda_B)},$$

and where $h(\cdot)$ is uniformly decreasing. The Pareto distribution satisfies these criteria, as do many other distributions.

A.3 Proofs Not Included in Main Text

This section contains proofs that are omitted from the main text.

Proof of Proposition 3 (Comparative statics). Define

$$\tilde{\Delta}(\hat{K}) = \lambda_A (m_{F,A} m_{I,A})^{\theta-1} [m_{F,A} + \phi m_{I,A}] - \lambda_B (m_{F,B} m_{I,B})^{\theta-1} [m_{F,B} + \phi m_{I,B}],$$

so that at an interior equilibrium, $\tilde{\Delta}(\hat{K}) = \hat{K}$. An equilibrium is stable if $\tilde{\Delta}'(\hat{K}) < 1$. Consider an interior, stable, equilibrium. We prove that as $G_A, \lambda_A, F_A$ rise, $\hat{K}$ rises. We make clear which results require the parameter assumption that, $\theta > (1 - \theta) \left( 1 + \frac{G_A}{F_A + F_B} \right)$. This is a more restrictive assumption that $\theta > 1/2$. We show the $F_B$ comparative static is ambiguous and depends upon $\hat{K}$. 

A.3
Preliminaries: conditions for $\frac{\partial u_A}{\partial m_{F,A}} > 0$. We have that

$$m_{I,A} = \frac{1}{\phi} \left( 1 + \frac{G_A}{F_A + H(\hat{K})F_B} \right),$$

and

$$\log u_A = (\theta - 1) \log m_{F,A} + (\theta - 1) \log m_{I,A} + \log (m_{F,A} + \phi m_{I,A}).$$

Then

$$\frac{\partial \log u_A}{\partial m_{F,A}} = (\theta - 1) \frac{1}{m_{F,A}} + \frac{1}{m_{F,A} + \phi m_{I,A}},$$

which is positive if:

$$m_{F,A} > (1 - \theta)(m_{F,A} + \phi m_{I,A}),$$

or

$$\theta > (1 - \theta) \left( 1 + \frac{G_A}{F_A + H(\hat{K})F_B} \right).$$

We assume the parameter restriction that

$$\theta > (1 - \theta) \left( 1 + \frac{G_A}{F_A + F_B} \right),$$

which ensures that $\frac{\partial u_A}{\partial m_{F,A}} > 0$.

Preliminaries: proof that $\frac{\partial u_A}{\partial m_{I,A}} > 0$. Next, we have that

$$\frac{\partial \log u_A}{\partial m_{I,A}} = (\theta - 1) \frac{1}{m_{I,A}} + \frac{\phi}{m_{F,A} + \phi m_{I,A}},$$

which is positive if:

$$\phi m_{I,A} > (1 - \theta)(m_{F,A} + \phi m_{I,A}),$$

or

$$\theta \left( 1 + \frac{G_A}{F_A + H(\hat{K})F_B} \right) > (1 - \theta),$$

which is always satisfied for $\theta > 1/2$.

Comparative statics for $G_A$. Given these results, it follows that $\frac{\partial u_A}{\partial G_A} > 0$. This because $m_{I,A}$ is increasing in $G_A$ and $u_A$ is increasing in $m_{I,A}$. Then, we have that

$$\frac{\partial \hat{K}}{\partial G_A} = -\frac{\partial (\hat{\Delta}(\hat{K}) - \hat{K})}{\partial \hat{K}} \frac{\partial u_A}{\partial G_A}$$

Since $\hat{\Delta}'(K) - 1 < 0$ (stability) and $\frac{\partial u_A}{\partial G_A} > 0$, we have that $\frac{\partial \hat{K}}{\partial G_A} > 0$. 

A.4
Comparative statics for $\lambda_A$. A similar argument follows for $\lambda_A$: $u_A$ is increasing in $\lambda_A$ and thus $\frac{\partial K}{\partial \lambda_A} > 0$.

Comparative statics for $F_A$. For $F_A$, note that $m_{F,A}$ and $m_{I,A}$ are increasing in $F_A$. Further, $U_A$ is increasing in $m_{F,A}$ and $m_{I,A}$ under the more stringent restriction on $\theta$. Thus it follows that $\frac{\partial K}{\partial F_A} > 0$.

Comparative statics for $F_B$. Last, consider $F_B$. All of the masses are increasing in $F_B$, and both $u_A$ and $u_B$ are increasing in the masses. Thus the sign of the comparative static depends on which effect dominates, and we get the ambiguous result. As $\hat{K}$ goes to $\infty$ and $H(\hat{K})$ goes to one, the effect on $u_A$ will dominate and hence increases in $F_A$ leads to a rise in $\hat{K}$.

\[ \square \]

Proof of Proposition 4 (Asset pricing). The first part of the proposition requires us to prove that

\[ \frac{\partial (P_{0, A} - \beta^2)}{\partial \hat{K}} > 0. \] (A.23)

Start by noting that

\[ \log(P_{0, A} - \beta^2) = \log \lambda_A + \theta \log m_{F,A} + (\theta - 1) \log m_{I,A} + \text{constant}. \] (A.24)

We differentiate this with respect to $H(\hat{K})$:

\[ \frac{\partial \log(P_{0, A} - \beta^2)}{\partial H} = \theta \frac{\partial \log m_{F,A}}{\partial H} + (\theta - 1) \frac{\partial \log m_{I,A}}{\partial H} \]

\[ = F_B \left( \frac{\phi \theta}{m_{F,A}} + \frac{\theta - 1}{m_{I,A}} \right) \] (A.25)

This is positive as long as

\[ \frac{\theta}{1 - \theta} > \frac{F_A + H(\hat{K})F_B}{G_A + F_A + H(\hat{K})F_B}, \] (A.27)

which always holds given that $\theta > \frac{1}{2}$. This implies that $\frac{\partial \log(P_{0, A} - \beta^2)}{\partial H} > 0$. Since $H(\hat{K})$ is a monotone increasing function of $\hat{K}$, we then obtain the desired result in equation (A.23). The second part of the proposition follows immediately from (A.24) given that $\theta - 1 \leq 0$.

\[ \square \]

Proof of Proposition 5 (Cross-sectional complementarities). Define

\[ \hat{\Delta}^+(\hat{K}^+) = \lambda_A m_{F,A}^\theta m_{I,A}^{\theta - 1} - \lambda_B m_{F,B}^\theta m_{I,B}^{\theta - 1}, \] (A.28)

\[ \hat{\Delta}^-(\hat{K}^-) = \lambda_A \phi m_{F,A}^{\theta - 1} m_{I,A}^\theta - \lambda_B \phi m_{F,B}^{\theta - 1} m_{I,B}^\theta. \] (A.29)

where now the masses are given by

\[ m_{F,A} = \phi \left[ F_A + H(\hat{K}^-)F_B^- \right], \quad m_{F,B} = \phi \left[ 1 - H(\hat{K}^-) \right] F_B^- \] (A.30)
and
\[ m_{I,A} = F_A + G_A + H(\hat{K}^+) F_B^+ , \quad m_{I,B} = G_B + \left[1 - H(\hat{K}^+) \right] F_B^+. \] (A.31)

Consider an interior equilibrium. The expression (A.28), holding \( \hat{K}^+ \) fixed, is strictly increasing in \( \hat{K}^- \) in an interior equilibrium, and weakly increasing in an equilibrium with no entry. Conversely, holding \( \hat{K}^- \) fixed, expression (A.29) is strictly increasing in \( \hat{K}^+ \) in an interior equilibrium, and otherwise weakly increasing. This yields the desired result.

**Proof of Proposition 6 (Sovereign incentives).** We start from the sovereign’s objective function, which for a given country \( j \) has the form
\[ W_j = G_j C_j + F_j U_j(G_j), \] (A.32)
where \( C_j \) is the convenience yield and \( U_j \) is average utility for \( j \)-firms. We take existing debt to be inframarginal, in the sense of earning the convenience yield set at issuance: therefore, for an infinitesimal perturbation, \( C_j \) in the expression above does not depend on \( G_j \) (i.e., \( \frac{\partial C_j}{\partial G_j} \)). We want to show that the leader country in a dominance equilibrium (\( A \)) has a stronger incentive to increase sovereign debt issuance \( G_j \) than the follower country (\( B \)), meaning that
\[ \frac{\partial W_A}{\partial G_A} > \frac{\partial W_B}{\partial G_B}. \] (A.33)

We begin by proving this result in the limit \( H(\hat{K}) \to 1 \). We start by noting that one can express the firm utility gradients as the sum of two components. A first component holds \( \hat{K} \) constant, while the second component captures the additional effect due to firm entry:
\[ \frac{\partial F_j U_j}{\partial G_j} = \nabla_{\text{No entry}} F_j U_j + \nabla_{\text{Entry terms}} F_j U_j. \] (A.34)

The relative scale of these two components depends on the entry gradient \( \frac{\partial H(\hat{K})}{\partial G_j} \). The component \( \nabla_{\text{No entry}} F_j U_j \) is multiplied by a scaling factor of one, while the component \( \nabla_{\text{Entry terms}} F_j U_j \) has scale \( F_B \frac{\partial H(\hat{K})}{\partial G_j} \).

It is clear then that the relative scale of the two terms depends on the level of entry \( H(\hat{K}) \). We have that:
\[ F_B \frac{\partial H(\hat{K})}{\partial G_j} = F_B h(\hat{K}) \frac{\partial \hat{K}}{\partial G_j}. \] (A.35)

As long as the distribution \( H(\cdot) \) has a finite mean (which the Pareto form used in our parametric analyses does, for \( \alpha > 1 \)), this tends to zero in the limit \( H \to 1 \) (or equivalently, \( \hat{K} \to \infty \)):\(^2\)
\[ \lim_{H \to 1} F_B \frac{\partial H(\hat{K})}{\partial G_j} = 0. \] (A.36)
\(^2\)To see this, consider that \( \lim_{\hat{K} \to \infty} \frac{\partial}{\partial \hat{K}} \int_{\hat{K}}^{\hat{K}'} KH'(K) dK = 0. \)
Therefore for large $H$, the component $\nabla_{F_j\mu_j}^{\text{Entry terms}}$ becomes negligible relative to the component $\nabla_{F_j\mu_j}^{\text{No entry}}$, and we can use the following approximation:

$$\frac{\partial F_j\mu_j}{\partial G_j} \approx \nabla_{F_j\mu_j}^{\text{No entry}},$$

(A.37)

which becomes exact as $H \to 1$.

Allowing for marginal pricing on corporate debt issuances, and using the approximation above, we then have that

$$\frac{\partial F_A\mu_A}{\partial G_A} = \phi \lambda_A F_A m_{F,A}^{\theta-1} m_{I,A}^{\theta-2} \left[ \theta G_A + (2\theta - 1)(F_A + H(\hat{K})F_B) \right] > 0,$$

(A.38)

where the inequality follows from the fact that $\theta > \frac{1}{2}$. We also have that

$$\frac{\partial F_B\mu_B}{\partial G_B} = \lambda_B \left( G_B + (1 - H(\hat{K}))F_B \right)^{\theta-2} \left( \phi F_B (1 - H(\hat{K})) \right)^{\theta} \left[ \theta G_B + F_B (H(\hat{K}) - 1 + 2\theta - 2\theta H(\hat{K})) \right] - \frac{\partial C(\hat{K})}{\partial G_B},$$

(A.39)

where $C(\hat{K})$ summarizes the foreign currency issuance costs paid by firms. In the limit $H \to 1$, these become:

$$\lim_{H \to 1} \frac{\partial F_A\mu_A}{\partial G_A} = \phi \lambda_A F_A m_{F,A}^{\theta-1} m_{I,A}^{\theta-2} \left[ \theta G_A + (2\theta - 1)(F_A + F_B) \right] > 0,$$

(A.40)

$$\lim_{H \to 1} \frac{\partial F_B\mu_B}{\partial G_B} = -\lim_{H \to 1} \frac{\partial C(\hat{K})}{\partial G_B},$$

(A.41)

Next, note that

$$\frac{\partial C(\hat{K})}{\partial G_B} = \frac{\partial}{\partial G_B} \int_{\hat{K}}^{K} KH'(K)dK = H'(K) \frac{\partial \hat{K}}{\partial G_B},$$

(A.42)

which tends to zero for $H \to 1$ for distributions $H(\cdot)$ with a finite mean:

$$\lim_{H \to 1} \frac{\partial C(\hat{K})}{\partial G_B} = 0.$$

(A.43)

It follows that

$$\lim_{H \to 1} \frac{\partial F_B\mu_B}{\partial G_B} = 0.$$

(A.44)

We can then conclude that, in the limit,

$$\frac{\partial F_A\mu_A}{\partial G_A} > \frac{\partial F_A\mu_B}{\partial G_B}.$$

(A.45)

The gradient of the seignorage component of the sovereign’s objective is simply the convenience

A.7
yield itself, independently of \(H(\hat{K})\), given inframarginal pricing of the existing debt:

\[
\frac{\partial G_j C_j}{\partial G_j} = C_j. \tag{A.46}
\]

Therefore as long as \(C_A > C_B\), we obtain that \(\frac{\partial W_A}{\partial G_A} > \frac{\partial W_B}{\partial G_B}\) in the limit \(H \to 1\) as desired.

These limiting results imply that for any real value \(\varepsilon > 0\), there exists a value \(H^\dagger = H(\hat{K}^\dagger)\) such that for all \(H > H^\dagger\), and hence for all \(\hat{K} > \hat{K}^\dagger\), we obtain that

\[
|\nabla_{F_j \hat{u}_j} \text{Entry terms}| < \varepsilon \quad \forall \ j, \tag{A.47}
\]

and

\[
\nabla_{F_A \hat{u}_A} \text{No entry} - \nabla_{F_B \hat{u}_B} \text{No entry} + \varepsilon > 0. \tag{A.48}
\]

By setting \(\varepsilon\) arbitrarily small, we obtain the desired result in the proposition.

**Proof of Proposition 7 (Normative implications).** Global welfare is \(W = W_A + W_B\). Call \(K^*\) the global welfare optimizing \(K\). We wish to show that \(K^* > \hat{K}_{\text{max}}\) in the case \(\theta = 1\). It is sufficient to show that

\[
\frac{\partial W}{\partial K} \bigg|_{K=\hat{K}} > 0. \tag{A.49}
\]

We have,

\[
\frac{\partial W_A}{\partial K} = G_A \lambda_A (\phi F_B h) + F_A \lambda_A 2(\phi F_B h), \tag{A.50}
\]

and,

\[
\frac{\partial W_B}{\partial K} = -G_B \lambda_B (\phi F_B h) - (1 - H) F_B \lambda_B 2(\phi F_B h) - \lambda_B (m_{F,B} + \phi m_{I,B}) h F_B + \lambda_A (m_{F,A} + \phi m_{I,A}) h F_B - \hat{K} h F_B + F_B H \lambda_A 2(\phi F_B h).
\]

Notice that the middle line here is equal to zero: it is the equilibrium condition that determines \(\hat{K}\). Then,

\[
\frac{\partial W}{\partial K} \bigg|_{K=\hat{K}} = \{G_A \lambda_A - G_B \lambda_B + 2F_A \lambda_A - 2F_B \lambda_B + 2F_B H \lambda_A + 2F_B H \lambda_B\} (\phi F_B h), \tag{A.51}
\]

which is positive in the case of \(A\) dominance \((G_A > G_B, \lambda_A \geq \lambda_B)^3\).

**A.4 Derivations for the Model With Aggregate Risk**

This section provides additional derivations for the extended version of the model with aggregate risk that we introduce in Section 4.5. We analyze this extended version of the model in the \(\theta = 1\)

\[3\]We also need \(W\) to be globally concave for this approach to be valid. It is straightforward to differentiate \(\frac{\partial W}{\partial K}\) and show that the second derivative is negative as long as \(h' < 0\), which the Pareto distribution satisfies.
case. To start, we consider the equilibrium at time $t_1$ in state $\omega$. The masses of liquidity providers are

$$m_{\omega I,j}^\omega = G_{\omega j}^A + F_j \quad (A.52)$$

where we recall that we allow for state-contingency in the supply of government assets $G_{\omega}^A$, but we assume that $F_j$ is not state-contingent. Correspondingly, the masses of liquidity demanders are

$$m_{\omega F,j}^\omega = \phi_{\omega F,j}. \quad (A.53)$$

The two-sided match probabilities are then

$$\alpha_{\omega F,j}^\omega = \lambda_j m_{\omega I,j}^\omega \quad P(\text{Buyer finds a seller})$$
$$\alpha_{\omega I,j}^\omega = \lambda_j m_{\omega F,j}^\omega \quad P(\text{Seller finds a buyer}). \quad (A.54)$$

The surplus from a match remains $1 - \beta$, independent of the aggregate state. The date $t_0$ price of the private bond is therefore

$$P_{0,j} = E \left[ \alpha_{\omega F,j}^\omega \beta + (1 - \eta)(1 - \beta) \right] + \frac{(1 - \alpha_{\omega I,j}^\omega)\beta^2}{P(\text{Not Matched}) \times \text{PV of Profit}}, \quad (A.55)$$

which we rewrite as

$$P_{0,j} = \beta^2 + (1 - \eta)\lambda_j E[m_{\omega F,j}^\omega] \beta(1 - \beta). \quad (A.56)$$

Firm utility at date $t_0$ is

$$u_{i,j}^F = P_{0,j} + \beta E[\phi_{\omega}^\omega m_{\omega F,j}^\omega] \eta(1 - \beta), \quad (A.57)$$

which, substituting in for $P_{0,j}$, becomes

$$u_{i,j}^F = \beta^2 + (1 - \eta)\lambda_j E[m_{\omega F,j}^\omega] \beta(1 - \beta) + \beta \lambda_j E[\phi_{\omega}^\omega m_{\omega I,j}^\omega] \eta(1 - \beta). \quad (A.58)$$

As before, we take the case $\eta = \frac{1}{2}$ and rewrite:

$$u_{i,j}^F - \beta^2 = \frac{1}{2} \lambda_j \beta(1 - \beta) \left( E[\phi_{\omega}^\omega m_{\omega F,j}^\omega] + \phi_{\omega}^\omega m_{\omega I,j}^\omega \right). \quad (A.59)$$

This expression is similar to that of the non-stochastic case, except that the masses are now stochastic.

We next substitute in for the masses and rewrite:

$$u_{i,j}^F - \beta^2 = \frac{1}{2} \lambda_j \beta(1 - \beta) \left( E[\phi_{\omega}^\omega(2F_j + G_{\omega}^A)] \right), \quad (A.60)$$

$$= \frac{1}{2} \lambda_j \beta(1 - \beta) \left( E[\phi_{\omega}^\omega(2F_j + E[G_{\omega}^A])] + cov[\phi_{\omega}^\omega, G_{\omega}^A] \right). \quad (A.61)$$

A.9
This yields the indifference condition presented in the main text:

\[
\lambda_A \left( \mathbb{E}[\phi_\omega](2(F_A + H(\hat{K})F_B) + \mathbb{E}[G_A^\omega]) + \text{Cov}[\phi_\omega, G_A^\omega] \right) - \hat{K} = \lambda_B \mathbb{E}[\phi_\omega] \left( 2(1 - H(\hat{K}))F_B + G_B \right).
\]  

(A.62)

### A.5 Investments in Private Revenue Pledgeability

We consider an extension of the model in which the sovereigns can invest in country-specific financial innovation which improves the pledgeability of firms’ revenue streams. We do this in the case where \( \theta = 1 \). In this extended version of the model, when firms apply for a bond issuance, they find out whether their project revenues are fully pledgeable (probability \( \rho_j \)) or non-pledgeable (probability \( 1 - \rho_j \)). The borrowing ability is idiosyncratic, so that \( \text{ex ante} \) a given firm is able to pledge its revenues with probability \( \rho_j \), and the law of large numbers applies across firms. After firms decide the currency in which to issue their debt, they discover their pledgeability, and if they choose to borrow they incur the fixed cost \( K_i \), as in the basic model. Thus, \( \rho_j \) captures the pledgeability of firm revenues in country \( j \). We set \( \theta = 1 \) for illustrative purposes.

The expected utility of borrowing in country \( j \) is then proportional to

\[
\rho_j \lambda_j [m_{F,j} + \phi m_{I,j}],
\]  

(A.63)

and the equilibrium condition for the marginal firm is now:

\[
\rho_A \left[ \lambda_A (m_{F,A} + \phi m_{I,A}) - \hat{K} \right] = \rho_B \left[ \lambda_B (m_{F,B} + \phi m_{I,B}) \right].
\]  

(A.64)

Increasing \( \rho_A \) increases the benefits of issuing in currency \( A \) in equation (A.64), thereby requiring the equilibrium \( \hat{K} \) to adjust to a higher value. As in the case with the government’s commitment technology, the incentive to invest is large in the dominant country, and moreover these stronger incentives are further reinforced by the endogenous increase in entry caused by the investments in private pledgeability:

\[
\frac{\partial W_A}{\partial \rho_A} > \frac{\partial W_B}{\partial \rho_B}, \quad \frac{\partial^2 W_A}{\partial \rho_A \partial \hat{K}} > 0, \quad \frac{\partial \hat{K}}{\partial \rho_A} > 0.
\]  

(A.65)

Proof. The equilibrium masses of firms are:

\[
m_{F,A} = \rho_A \phi \left[ F_A + H(\hat{K})F_B \right], \quad m_{F,B} = \rho_B \phi \left[ 1 - H(\hat{K}) \right] F_B,
\]  

(A.66)

while the masses of liquidity suppliers are,

\[
m_{I,A} = G_A + \rho_A F_A + \rho_A H(\hat{K})F_B, \quad m_{I,B} = G_B + \rho_B \left[ 1 - H(\hat{K}) \right] F_B.
\]  

(A.67)
Consider the objectives for country A and B. We have that

\[ W_A = G_A(\lambda_A m_{F,A}) + \rho_A F_A \lambda_A (m_{F,A} + \phi m_{I,A}) \]
\[ = G_A \lambda_A \rho_A \phi \left[ F_A + H(\hat{K}) F_B \right] + \rho_A F_A \lambda_A \left( \rho_A \phi \left[ F_A + H(\hat{K}) F_B \right] + \phi (G_A + \rho_A F_A + \rho_A H(\hat{K}) F_B) \right) \]

and,

\[ W_B = G_B \lambda_B m_{F,B} + \rho_B F_B (1 - H(\hat{K})) \lambda_B (m_{F,B} + \phi m_{I,B}) + U_{B \rightarrow A} \]
\[ = G_B \lambda_B \rho_B \phi \left( 1 - H(\hat{K}) \right) F_B \]
\[ + \rho_B F_B (1 - H(\hat{K})) \lambda_B \left( \rho_B \phi \left( 1 - H(\hat{K}) \right) F_B + \phi (G_B + \rho_B \left( 1 - H(\hat{K}) \right) F_B) \right) + U_{B \rightarrow A} \]

It is straightforward to see that:

- \( \frac{\partial W_A}{\partial \rho_A} > \frac{\partial W_B}{\partial \rho_B} \) since \( G_A > G_B \), \( \lambda_A \geq \lambda_B \) and \( F_A \geq F_B \).
- \( \frac{\partial^2 W_A}{\partial \rho_A \partial \hat{K}} > 0 \) since \( \frac{\partial m_{F,A}}{\partial \hat{K}} \) and \( \frac{\partial m_{I,A}}{\partial \hat{K}} \) are positive and \( \frac{\partial \hat{K}}{\partial \rho_A} \) is positive.

\[ \square \]
B  Further Historical Details

This section provides additional details which supplement our discussion of the various historical episodes.

B.1  Bank of Amsterdam

The Dutch florin created by the Bank of Amsterdam in 1609 was the first global currency. For much of history, transactions and debts around the world were primarily settled in metallic coins. However, hundreds of domestic and foreign varieties existed, and using them entailed large transaction costs such as transportation, insurance, and assayance.\(^4\) The difficulty of enforcing quality created incentives to debase the currency and reduce the circulating supply of high-quality coins. These costs compounded the difficulty of coordinating on the coins that were valid for settling a debt. While negotiable credit instruments such as the bill of exchange reduced the need to transfer coins, they still required a unit of denomination and so settlement ultimately relied on an uncertain supply of physical assets.

The Bank of Amsterdam was chartered by the City of Amsterdam to provide a high quality standardized currency that would reduce settlement frictions. The Bank primarily did so by creating a currency that existed on its ledgers (“bank florin”) that was backed by coin and could be transferred across accounts freely. The City of Amsterdam initialized the pool of florin available for settlement by requiring that all large bills of exchange drawn and/or payable in Amsterdam had to be settled at the Bank, i.e., denominated in florin.\(^5\) Relative to the uncertain supply of specific metal coins in circulation anywhere else in the world, the Bank provided florin the advantage that there would be a ready supply for payments in Amsterdam.

Bank accounts were freely provided to anyone, and florins were credited to accounts for deposits of recognized coins. These coins backed the florins that could be withdrawn as current guilders in a narrow bank model. Since the Bank charged a fee for withdrawals, it was usually less costly to trade florin for current guilder in a secondary “open market.”\(^6\) In that market, the *agio* was the market exchange rate between the bank florin and the current guilder.

Rotterdam, a neighboring mercantile city, also created its own exchange bank modeled after the Bank of Amsterdam. While the two institutions maintained separate balance sheets, Rotterdam adopted Amsterdam’s *agio* because merchants preferred florin (Van der Borght, 1896, p. 209). Rotterdam provided a system where all deposits and withdrawals of guilders were made allowing for the Amsterdam *agio* and thus used Amsterdam’s florin as the unit of account (Carey, 1818, 1918).

\(^4\)An ordinance in the Dutch Republic from 1606 officially recognized 25 gold and 14 silver trade coins from 35 domestic mints, but many more varieties circulated, and the Republic officially published exchange rates for almost 1000 coins (Roberds and Velde, 2016, p. 344).

\(^5\)The first ordinance in February 1609 applied to bills over 600 guilders; in 1642 year this was revised to include bills over 300 guilders. As a result, all merchants kept an account at the Bank, and the Bank maintained two to three thousand accounts at any given time (Van Dillen, 1934, p. 107).

\(^6\)The withdrawal fee of 1.5% covered the costs that the Bank of Amsterdam incurred to mint current guilders for deposits of inferior coins (judged by their metallic content).
p. 369). Rotterdam also conceded to provide current accounts, which were the primary means for merchants to access florin by way of guilders and much more heavily used than its own bank money (Van der Borght, 1896, p. 210). In addition, the Bank of Rotterdam, despite requiring large bills of exchange to be settled in its own bank florin, also settled bills payable in florin in Amsterdam. In these ways, Rotterdam provided access to florins to the extent possible given its separate balance sheet.

In 1683, the Bank also introduced a receipts technology that operated like a modern day repurchase agreement. The Bank of Amsterdam advanced florin for short-term deposits of specie and metal bars. Depositors were issued a receipt, negotiable and renewable with an initial maturity of six months, for the right to withdraw the specific metal they deposited. This technology broadened the set of assets that could be converted into florin beyond the original set of trade coins, and it was beneficial for both the Bank of Amsterdam and for private parties. The former gained from the metal deposits, which became part of the Bank’s assets if the receipt expired, and the latter was able to obtain florin for settlement without needing to convert them into eligible trade coins for deposit at market value.

Following the introduction of the repo facility, the quantity of florins at the Bank of Amsterdam doubled from approximately eight million to sixteen million from the mid 17th to the beginning of the 18th century (Quinn and Roberds, 2014b). These complementary innovation forces are also at play in the rise of Amsterdam. For instance, the receipts technology (i.e., repurchase facility) that created florin claims out of raw specie was only introduced in 1683, almost seven decades after the Bank’s establishment. The fact that florin balances doubled after its introduction is indicative of the force identified in our model that increases in \( \lambda \) generate entry (Quinn and Roberds, 2014b).

After a century of dominance, the Bank of Amsterdam eventually became a victim of its own success. Intermediation in bank florins was profitable for the Bank, and it routinely turned over its wealth to the City of Amsterdam, leaving it with little capital buffer. It also made advances to the Dutch East India Company (VOC), which eventually led to runs on bank florin after the VOC came close to failure following the fourth Anglo-Dutch War in 1784. The French invasion in 1795 led to a drop in the agio to -14%, after which it never fully recovered, and eventually the Bank was formally dissolved in 1819.

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7 There was a large secondary market in receipts. Receipts could be redenominated in smaller face values, and they were renewable by paying the withdrawal fees. Withdrawal fees with receipts (0.125% for silver and 0.25% for gold) were much lower than that for current guilders (1.5%) because the Bank did not need to mint guilders to meet withdrawal demands. Around this time it appears the Bank of Amsterdam eliminated the right to withdraw from its accounts, which has led some authors to argue that the florin was an early fiat currency (Quinn and Roberds, 2014b).

8 Given the wide variety of specie circulating, the demand and supply for specific coins varied significantly, and market prices were usually in flux. The receipts technology made it possible to transact on the Bank’s mandated value for the specie while retaining the ability to withdraw and sell at a future date when prices rose. It also supported a large trade in precious metals since the freely-traded receipts were equivalent to advances on pledges of the underlying metals.
B.1.1 Dutch versus Spanish Trade

The figures on Spanish trade per capita and the population are taken from Ortiz-Ospina et al. (2018) and Allen (2003) on p. 438 (Table A1) respectively. Data are available for four years (1600, 1700, 1750 and 1800). Dutch trade comes from Zanden and Leeuwen (2018) survey. We extract the four data points to compute Spanish trade relative to Dutch trade in the 17th – 19th centuries. Pound-guilder exchange rates are taken from Denzel (2017) (Figure 28.1).

Table B.I: Estimates to compute Spanish trade

<table>
<thead>
<tr>
<th>Year</th>
<th>Holland (in million guilder)</th>
<th>Trade Spain (in £ per capita)</th>
<th>Population Spain</th>
<th>Exchange rate £-guilder</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
<td>13.462</td>
<td>.18907407</td>
<td>8,700,000</td>
<td>0.095</td>
</tr>
<tr>
<td>1700</td>
<td>12.152</td>
<td>.31009123</td>
<td>8,600,000</td>
<td>0.1</td>
</tr>
<tr>
<td>1750</td>
<td>10.221</td>
<td>.57871836</td>
<td>9,600,000</td>
<td>0.095</td>
</tr>
<tr>
<td>1800</td>
<td>16.241</td>
<td>.48912659</td>
<td>13,000,000</td>
<td>0.09</td>
</tr>
</tbody>
</table>

In order to compare Spanish trade to Dutch trade volumes, we need the estimates for Spanish trade in guilder. Thus, we multiply the Spanish trade in £ per capita by Spanish population and divide it by the £–guilder exchange rate to arrive at the final figures below.

Table B.II: Trade volumes (in million guilder)

<table>
<thead>
<tr>
<th>Year</th>
<th>Holland</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
<td>13.462</td>
<td>17.315</td>
</tr>
<tr>
<td>1700</td>
<td>12.152</td>
<td>26.667</td>
</tr>
<tr>
<td>1750</td>
<td>10.221</td>
<td>58.481</td>
</tr>
<tr>
<td>1800</td>
<td>16.241</td>
<td>70.651</td>
</tr>
</tbody>
</table>

B.2 Great Britain

This initial point established the pound as a dominant currency with substantial entry from foreign sovereigns. The institutional developments in the legal structure of bills of exchange and the role of the Bank of England also reflect investments in overall market liquidity $\lambda_A$. These investments continued throughout the 19th century as dominance engendered entry, liquidity, and increased incentives to innovate.

The Bank of England was a key institution in the London money market, founded in 1694 as a note-issuing private corporation that was granted several privileges in return for raising and administering the Crown’s debt. During the early part of its history, the Bank competed with

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9The vast majority of the sovereign debt issued in London after the Napoleonic Wars were by Latin American and other European nations rather than the British colonies (Meyer et al., 2022).

10Like most currencies during this era, the pound sterling referred to specific metallic coins, and obligations denominated in sterling were contracted to be repaid in those coins. However, coins were inconvenient for
other private banks to increase its note circulation and raise its profits. In this respect, the Bank was like any other private firm that was incentivized to issue safe debt in order to benefit from the yield premium in equation (A.56). It was very successful in establishing a sound reputation for its notes, and by the late 18th century, Bank notes became synonymous with the pound sterling (Thornton, 2017).

In 1833 and subsequently 1844, the de facto equivalence between the pound and Bank notes became de jure with passage of the Bank Notes Act and the Bank Charter Act respectively. The former made Bank notes legal tender while the latter consolidated the entire note issuance onto the Bank of England’s balance sheet where it was fully backed in gold reserves above the allowed fiduciary issue. The full note circulation of the Bank of England therefore officially contributed to the supply of $G_A$ following the 1840s.

A second important innovation was the legal codification of the contractual terms for bills of exchange, which coordinated the market on the terms of borrowing and the procedures for default. Bills of exchange were the primary London money market instrument, and each time one was traded (“discounted”), the seller guaranteed (“endorsed”) the bill. These endorsements were legally equivalent to being the original borrower, and so each endorser was equally liable. The generality of these conditions were constantly tried at court and established a strong legal precedent. These laws reduced the information sensitivity of bills and collectively raised the safety and liquidity of all bills of exchange with multiple endorsers, regardless of the idiosyncratic characteristics of the ultimate borrower. Thus, these innovations made private debt money-like in the sense of Dang et al. (2017).

A third notable institution was the Bank of England’s role as a credible and reliable lender of last resort to the financial sector. During the banking crises of 1847, 1857, and 1866, the Bank obtained permission from the Treasury to suspend the Bank Charter Act in order to meet all

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11The Banking Act of 1826 required the Treasury to monitor the amount of small-denomination notes issued by the Bank following the 1825 crisis in which the Bank was seen to have abused its monopoly (Scammell, 1968, p. 132).

12Estimates of historical convenience yields in the era of British pound dominance include Chen et al. (2022) and Payne et al. (2022).

13The limits of the Bank’s note supply was therefore primarily governed by the gold reserves at the bank and secondarily by the government-determined fiduciary issue. Private bank notes already in circulation were allowed to remain, but no new notes could be issued, and banks lost their right when they merged. The Bank Charter Act could be suspended during financial crises when there was large demand for Bank notes.

14A Parliamentary report from 1837 describes the legal protections against default: “a holder of a bill of exchange can bring actions at one and the same time, against every party whose name is attached to it, and in the event of the failure of them all, can prove upon the estate of each for the full value of the bill” (Joplin, 1837, p. 17).

15An additional factor is that the Act of 1833 exempted short-term bills of exchange from the Usury Laws, which also expanded the market’s general willingness to hold them (Scammell, 1968).
demand for Bank notes. In fact, the Bank’s behavior during the crisis of 1866 was the basis for Bagehot’s rules for central banking (Bagehot, 1873). As a lender of last resort, the Bank provided liquidity at its discount window by converting private bills of exchange into pounds sterling, thereby de facto became a backstop to the private bills market. This backstop officially only applied to high-quality bills—those first guaranteed (“accepted”) by large merchant banks that held accounts at the Bank of England—but like all liquidity backstops, its existence reduced the occurrence of market freezes and increased the willingness of private firms to lend in all states. These forces together led issuers to prioritize denominating issues in sterling, thereby increasing the quantity of safe pound-denominated debt in the London money market.

The Bank of England acting as a lender of last resort was a major transition from its earlier history in the 18th century when discounting and note issuance was a profit-maximizing endeavor. At that point, the Bank’s discount window followed the market and became similarly unavailable during downturns and crises. As the London money market deepened and the pound sterling gained dominance in the 19th century, the Bank increasingly took on a more formal role within the government. This was despite the fact that it remained privately owned by stockholders until 1944 and run by Governors and Courts of Directors that primarily stemmed from the merchant and banking classes (Cassis, 1994, p. 85). The Bank’s transitioning role in the money market given its dual identities reflects how the benefits of agglomeration accrued to both the government and to the private sector, all embodied in a single institution.

One final development during this period that contributed to maintaining the dominant equilibrium is the growth of international banking, which facilitated access to the pound sterling in locations around the world. British overseas banking institutions generally followed the business model of issuing deposits and shares domestically while lending via bills of exchange payable in London in their branches abroad. As in Amsterdam, the short term commercial bill became the dominant credit instrument internationally, with payments settled in London even for transactions that did not involve Great Britain. The network of British banks increased the likelihood that

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16Since the Bank Charter Act limited the supply of Bank notes to the Bank’s gold reserve, suspending it (and therefore the gold standard) was the only way to ensure they could meet demand. Even when the gold reserve was high, the presence of a limit reduced liquidity in the market. It is worth noting that obtaining permission to suspend the Bank Charter Act was sufficient, and Great Britain did not actually suspend the gold standard during this period.

17The high quality bills eligible at the Bank of England became a class of their own, and the financial press throughout this period reported the rates on “Bank” bills separately from “trade” bills (Xu, 2022).

18Scammell writes, “All in all the discounting of bills by the Bank in the early 19th century must be seen primarily as a prosperous business of the Bank and only very secondarily as a manifestation of credit policy,” (Scammell, 1968, p. 144). For example, during the 1797 crisis, Parliament assumed the role of being a liquidity provider by issuing exchequer (treasury) bills to the market (Thornton, 2017, p. 98). The subsequent crisis in 1825 provides a microcosm into the transition that took place. Early in the year, the Bank of England closed its discount window because it anticipated a financial market downturn. This action in itself ”created an atmosphere of misgiving and potential crisis,” (Scammell, 1968, p. 131). However, when the crisis peaked in November with numerous failures in London, the Bank reversed its earlier decision and made discounts and advances on government securities and private bills. Thereafter starting in 1830, it allowed bill brokers to access the discount window for the first time, after recognizing that these institutions were important conduits of liquidity.

19For example, “the bill on London enabled the banks [...] to finance a large share of international trade
foreign firms could hold pound obligations (or equivalently receive part of their profits in pounds), which in the context of the model we view as equivalent to reducing the cost of foreign currency issuance $K_i$, whether through a reduction of underlying FX exposures or via a reduction in the fixed cost of debt underwriting. Reducing this cost increases the mass of firms for which issuing in the foreign currency is profitable, as shown in equation (A.2).\textsuperscript{20}

### B.3 Comparing British Empire and US GDP

Figure B.1 plots the US, UK, and British Empire shares of global GDP from 1850 until today. This figure shows that the US overtakes UK GDP in the 1860s, and that the US overtakes the British Empire in 1901. These transitions in size occurred before the very beginning of the period of dollar dominance, which can be dated to the end of WWI at the very earliest, but is generally dated to the end of WWII.

![Figure B.1: Share of Global GDP](image)

**Notes:** We construct Figure B.1 using data from the Maddison Project Database (version 2020). Countries are counted as part of the British Empire using contemporary definitions. The UK includes Great Britain, Scotland, Wales, and Ireland. We include countries when they provide uninterrupted annual GDP data. The only country with interpolated data is India prior to 1884, in which missing GDP values were filled in based on the previously reported year. This interpolation method for India, chosen for illustrative reasons, avoids a significant spike in the Empire’s GDP between 1883 and 1884 when India consistently enters our sample. This approach does not affect the year the USA took over the British empire, which is 1901.

Incidentally, both the French and the Germans followed the British model, often with explicit reference to expanding their currencies abroad. For instance, Edward Hurley, in his arguments for the creation for the US Federal Reserve System wrote, “The logical ambition of the German commercial policy is naturally to enthrone the Mark in the estimation of the world until it need pay no deference to the pound sterling.”

\textsuperscript{20}Regardless of whether that trade touched Britain’s shores,” (Orbell, 2017, p. 8), and “wines from France, coffee from Brazil, sugar from the West Indies, and silk from Hong Kong were paid alike with bills on London,” (Jenks, 1927, p. 69).
References for Online Appendix


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