Liquidity, Debt Denomination, and Currency Dominance

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Abstract

The international monetary system of the last four centuries has experienced the rise, persistence, and fall of specific currencies as the dominant unit of denomination in global debt contracts. We argue that a liquidity-based theory is necessary to explain this pattern. Firms issue debt that can be extinguished by trading their revenues for financial assets of the same denomination. When asset markets differ in their liquidity, as modeled via endogenous search frictions, firms optimally choose to denominate debt in the unit of the asset that is most liquid. Equilibria with a single dominant currency emerge from a positive feedback cycle whereby issuing in the more liquid denomination endogenously raises the benefits of that denomination. This feedback mechanism has historically been seeded by governments committed to the largest pool of liquid assets in the same denomination. Once a dominant currency emerges, the government hosting the currency endogenously invests more in the liquidity of its financial markets, leading to further entrenchment of that equilibrium. Our theory explains the historical experiences of the Dutch florin, the British pound sterling, the US dollar, and the transitions between them. We rationalize the current dollar-dominant international financial architecture and provide predictions about the potential rise of the Chinese renminbi.

Keywords: International Monetary System, Dollar Dominance, Liquidity, Corporate Debt.

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1 Introduction

The international monetary system has featured few but persistent dominant currencies in the denomination of financial contracts. The first dominant currency, the Dutch florin of the 17th century (Quinn and Roberds 2014b), was followed by the British pound sterling in the 19th century (Lindert 1969; Eichengreen 2005), which in turn was replaced by the US dollar in the 20th century. This paper posits a theory for why, among many possibilities for debt denomination, one currency emerges endogenously as dominant in equilibrium and is able to retain that dominance in global debt contracts for prolonged periods. Our theory proposes that debt market liquidity is a key component of the factors establishing currency dominance both historically and today.

We begin by observing that when borrowers such as firms issue debt, they must choose a unit of denomination for the contract. This choice determines the asset that the borrower must hand over to extinguish the debt at the time of settlement. The borrower, for example a farmer, could issue debt that is denominated in any arbitrary unit, such as bushels of wheat. In this case, upon maturity the farmer would be obliged to deliver bushels of wheat. But if bushels of wheat are difficult to come by—that is, if the wheat market is illiquid—this would be a costly decision. Perhaps a wheat farmer would issue such debt, but a manufacturer would not. The settlement benefits of asset market liquidity, which we model via endogenous search frictions (Duffie, Gârleanu and Pedersen 2005; Weill 2020), constitute the first key economic force in our model.

In today's international monetary system therefore, an appeal of issuing debt denominated in dollars is the large and liquid nature of the dollar money market that facilitates dollar settlement.[1] All issuers worldwide can see that there is a significant number of investors that own a substantial amount of dollar-denominated, short-term money market instruments such as Treasury bills, repos, or high-grade bank and firm debt. As a result, they benefit from being able to easily trade their revenue streams with these investors to acquire the assets necessary to settle their debt obligations.

The second key economic force is that debt issuance itself expands the demand for future settlement and therefore raises the returns to issuers who can supply assets that provide settlement benefits. For such issuers, it is therefore more beneficial to issue in a denomination in which the assets created are more likely to be used for settlement. Together, supply and demand for dollar debt are complementary, bootstrapping each other, leading to currency dominance.

The liquidity force in asset markets is the distinguishing factor that rationalizes the first historical dominant currency experience, that of the Dutch florin. During this period, Spain was the largest and wealthiest economy with the highest volumes of trade. However,

[1] Dollar dominance has many features including dollar-denominated debt outstanding in the world being outsized relative to the wealth or GDP share of the United States (see Cetorelli and Goldberg 2012; Bruno and Shin 2015b; McCauley, McGuire and Sushko 2015; Maggiori, Neiman and Schreger 2020).

Spain's financial markets were relatively undeveloped, and even though the Spanish *real de ocho* ("pieces of eight") were the most common coin in the world, the quantity and quality of these coins, like all other metallic coins, varied across time and space, limiting their settlement benefits. In contrast, the Bank of Amsterdam harnessed a financial technology to create a new currency called the "bank florin" as a denomination that existed purely on the Bank's ledgers (and hence was not subject to frictions associated with transport, insurance, and degradation), and which could be used to settle obligations via fast and free account transfers. Parties around the world, even those not transacting directly with Amsterdam, typically chose *florin* denomination over the currency of the largest economy at the time (Spain), reflecting the value of the florin's superior liquidity coming from its ledger-based settlement technology.[2]

We formalize this liquidity force for dominance by modeling a financial demand for liquidity, similar to Holmström and Tirole (1998), whereby assets are needed to settle debt obligations. At the same time, asset market liquidity depends endogenously via search frictions on the quantity of available liquid bonds, as in Duffie, Gârleanu and Pedersen (2005). The key decision we study is that of a private borrower (a firm) whose revenues are in a home currency and who must choose the denomination of its debt in either the home or foreign currency. Choosing to denominate in foreign currency entails exchange rate risk and costs associated with currency mismatch in the firm's revenues and liabilities. Firms weigh this cost against the two benefits of issuing in a more liquid foreign currency: the ease of settling its own obligations and the convenience yield (more favorable borrowing terms) that accrues for issuing safe debt that can be used as a means of settlement by others.

We begin by presenting theoretical arguments and historical evidence in support of the liquidity mechanism that we propose. First, our theory highlights the settlement aspect of denomination rather than the returns aspect. For example, in today's world, consider the payoffs of Swiss franc or Canadian dollar debt as an alternative to US dollar debt. These currencies have exchange rates with similar payoff characteristics as the dollar: they are safe, and they appreciate during periods of economic turmoil. Yet in terms of quantities, dollar-denominated debt is many orders of magnitude larger than debt denominated in these other currencies, and therefore dollars are the dominant choice for private firm debt denomination. Historically, the classical gold standard era provides similar examples. Currencies like the German mark and French franc provided a claim to the same underlying specie payoff as the British pound sterling, and yet there was much less foreign debt issuance in marks or francs relative to pounds. Our theory predicts more private debt issuance in the more liquid currency, which makes it possible to distinguish which currency, among several equally safe

[2]Bills of exchange were the primary financial instrument of early modern Europe, and florin-denominated bills payable at the Bank of Amsterdam has been described as "the premier means of international payment in Europe" during this period (Quinn and Roberds, 2019, p. 736). In addition, the "[Bank of Amsterdam was] the clearinghouse of world trade" even for non-Dutch trade, as was the case for the trade between England and Russia, which was conducted exclusively through payments in Amsterdam even though the goods themselves did not pass through Amsterdam (De Vries and Van der Woude, 1997, p. 87, 131).

ones, will become dominant.

Second, our theory focuses on asymmetries in financial market liquidity, emphasizing how these generate a unique dominant equilibrium. This asymmetry has historically stemmed from large differences in the float of safe short-term government-backed liabilities, as it does today, as emphasized by Caballero, Farhi and Gourinchas (2008). The government's commitment to these liabilities has taken various forms, whether through physical reserve holdings or via fiscal commitments. At the Bank of Amsterdam, the florin was backed by specie reserves, and confidence in the City of Amsterdam's commitment not to appropriate that specie was crucial for enabling it to implement its financial technology and to create a large float of florin. In contrast, Spain's history of serial defaults made it unable to provide such a commitment. Similarly, the pound sterling's dominance was established after Dutch losses in the Fourth Anglo-Dutch War caused a collapse in confidence in Amsterdam, and the British subsequently won the Napoleonic Wars. At that point, the British government's fiscal capacity allowed it to back the most liquid pool of safe assets, denominated in pounds. In the post-Bretton Woods period, the United States government has backed a large quantity of dollar-denominated short-term debt instruments in the form of Treasury bills, with more safe float than the government liabilities of alternative currencies.

Our model clarifies that the role of economic size in generating dominance is through its impact on financial market liquidity, but that size is not itself a fundamental determinant of dominance. Increasing the size of the private sector in the non-dominant country can in fact entrench dominance, rather than leveling the field, because it may be optimal for its firms to maintain issuance in the dominant currency, thereby increasing supply and liquidity in the dominant currency. In our model, as was often the case historically, the dominant currency need not be hosted by the country with the largest GDP. For example, the florin was dominant despite Amsterdam being several times smaller than Spain, and at the turn of the 20th century, the United States surpassed the entire British empire in economic size, but American firms continued to issue bonds in London in sterling rather than domestically in dollars. In line with our focus on asymmetry in financial market depth, the shift from sterling to the dollar only occurred after 1913, when the US increased its stock of government backed liabilities and the creation of the Federal Reserve system pooled different regional financial centers into a central market that helped enhance the dollar money market's liquidity.

Third, the role of international trade in our model, like that of economic size, is that it can reinforce a dominant equilibrium that already exists, but trade volumes themselves are not necessary to generate dominance, as we have seen in previous historical episodes. In the context of the theory, if firms receive their revenues in the foreign currency through trade invoicing, this will generate an additional source of asset-liability complementarity. While this additional assumption allows us to rationalize the joint observation of trade and finance in a dominant currency, the volume of trade is not a necessary condition for generating financial dominance. In addition, our financial liquidity theory explains why the volume of foreign

currency financial flows is not bound to the volume of flows in goods and indeed can be much bigger, as it is in today's dollar world.

Further, we argue that a financial liquidity theory is necessary to explain related domestic phenomena in which open-economy forces such as trade invoicing denomination are absent. In particular, while the primary purpose of our theory is to explain the patterns of debt denomination observed in the international economy, we also rationalize the prevalence of nominal, rather than real, debt denomination within a specific country. Debt denomination in real terms provides a consumption utility hedge against inflation, yet there is relatively little real denomination in private debt. Our theory explains that this phenomenon stems from government-committed securities also primarily being denominated in nominal terms. Indeed, the historical experience of the United States illustrates this argument: when the country was on the gold standard, and government debt was repaid in gold, private debt contracts also included a claim to gold. However, during the "greenbacks" era of the late 19th century when government debt was denominated in nominal fiat dollars, private bonds also switched to those nominal dollars, consistent with our model's predictions.

Having argued in favor of a liquidity-based mechanism for currency dominance, we next move on to outlining how our theory jointly rationalizes many additional features of the international monetary system. First, our model shows that the country hosting the dominant currency has a greater capacity and incentive to enhance financial market liquidity, thereby generating an additional source of complementarity that entrenches dominance and leads to the persistence of regimes. Consider a costly action that allows for an increase in government-backed safe assets—for example, improvements in the country's legal institutions or its military capacity to capture more resources. The convenience yield from safe debt issuance, which is seigniorage revenue for the government, is higher in the dominant currency. In addition, the incentive to undertake this action is lower for the non-dominant country since the marginal gain in seigniorage revenues is lower and additionally, liquidity expansion in the non-dominant currency does not benefit firms that have already chosen the dominant currency. It is therefore beneficial for the dominant country to invest more in such costly actions. This expansion allows for more debt issuance, which endogenously attracts more firms to the dominant currency. Historically, dominant country investments, for example in Amsterdam and the UK, have been directed towards financial and legal institutions as well as military capacity.

Second, by generating both settlement and convenience yield benefits to issuance, our theory explains why empirically, there is a wide variety of borrowers that issue debt in the dominant currency in the cross-section. In today's dollar world, some international firms have low credit ratings and hence their bonds are not sufficiently money-like to be used as settlement liquidity by other parties, but these borrowers still benefit from the settlement liquidity in the dollar money market. Other borrowers, say a safe international issuer such as the German government-backed supranational bank KfW, likely do not have as many settlement needs, but nonetheless issue dollar debt to capture the convenience yield. Our model therefore allows

us to explain the wide cross-section of firm issuance in the data, particularly by borrowers that do not earn a convenience yield on their debt.

Third, the dual issuance benefits in our model provide novel predictions for the impact of increasing the supply of safe government debt on the issuance incentives of private borrowers. We show that within a given dominant equilibrium, for a reasonable set of parameters, expanding the pool of safe government-backed liabilities crowds out safe private debt issuance, as has been empirically documented in Krishnamurthy and Vissing-Jorgensen (2012), while crowding in risky debt issuance. This result contrasts with other work that does not distinguish between different types of private debt or focuses solely on safe debt.

Fourth, our theory sheds light on normative aspects of liquidity provision arrangements in the international monetary system. The decision to denominate debt in the dominant currency carries a positive externality, as it improves market thickness for all other borrowers and lenders. As a result, the competitive equilibrium is not efficient: a global planner will want to subsidize even more denomination in the dominant currency. Socially beneficial coordination can be implemented with formal international arrangements, like the Bretton Woods system, in which the commitment devices backing a country's monetary liabilities (such as gold) are concentrated in a single country, while all other countries have no formal need for gold.

Finally, we extend the model to include default risk and liquidity provision policy tools. At least since the Bretton Woods arrangement, the US has committed to state-contingent expansions of dollar liquidity via central bank swap lines. The Federal Reserve provided these swap lines in several recent crises when global banks and foreign central banks exhibited large demands for dollars to settle their firms' liabilities. Our model shows that such expansions enhance dominance. We introduce aggregate shocks to firms' liquidity demand and allow the government to tailor liquidity supply to depend on the aggregate state. State-contingent policies that generate positive covariance between liquidity supply and liquidity demand, such as central bank swap lines during crises, make the dominant currency more attractive. By the same token, however, if liquidity supply falls in a high liquidity demand state—negative covariance—this can undercut a currency's appeal. This latter point helps explain why the euro has lost reserve currency ground to the dollar after the 2008 financial crisis (Maggiori, Neiman and Schreger, 2020), when Euro-area sovereign debt downgrades were followed by a sharp contraction in the international use of the euro.

Related Literature. Our paper most directly relates to work that explains the emergence of a dominant unit of account in financial contracts. All theories that generate dominance require a source of complementarity in agents' decisions. Doepke and Schneider (2017) presents a model of input-output chains of production where agents choose the denomination of contracts for goods purchases and sales. In this setting, it is efficient to avoid asset-liability mismatch that could lead to costly default by coordinating on a single denomination of these contracts.

Furthermore, if there is a sufficient quantity of government debt that agents hold as assets, then the unit of denomination of this debt will be chosen as the unit in private contracts in order to minimize balance sheet risk. The positive link between an ample supply of safe government liabilities and denomination decisions also occurs in our theory, but through a liquidity channel. Drenik, Kirpalani and Perez (2022) build on the same framework with unit-of-account considerations to study the interaction between the currency choice of private contracts and optimal monetary policy.

While we also keep the cost side of the debt denomination decision (i.e., asset-liability mismatch), our paper highlights new channels for the benefits of denominating financial contracts in the dominant currency, stemming from ease of settlement in financial assets and increasing returns in the provision of liquidity in a common denomination. There is an empirical literature showing that a wide variety of firms, including non-exporters, choose foreign currency debt denomination and are rarely fully hedged against exchange rate fluctuations (Verner and Gyöngyösi, 2020; Adams and Verdelhan, 2022). Our model of liquidity benefits that accrue to all firms rationalizes the wide cross-section of issuance and the phenomenon of firms choosing foreign currency exposure in equilibrium, both historically and today.

In the international setting, the literature has related the centrality of the US and the dollar to a range of forces.[3] Gopinath and Stein (2021) and Chahrour and Valchev (2022) focus on dollar trade invoicing where agents aim to reduce asset-liability mismatch: the need to finance imported goods leads to optimal coordination of liabilities in the same denomination. In Gopinath and Stein (2021), the complementarity is seeded and enhanced by a large quantity of dollar-invoiced trade between the US and the world, and in contrast, undercut by a large supply of US government-backed safe assets. In our model, the complementarity is enhanced by both factors. Caballero, Farhi and Gourinchas (2008) analyze asymmetries in the capacity to produce safe assets, while Farhi and Maggiori (2018) study settings with monopoly power in asset provision. He, Krishnamurthy and Milbradt (2019) analyze coordination motives in a debt rollover game, showing that these motives can be strengthened by a large float of US government debt. Maggiori (2017) and Gourinchas and Rey (2022) present models where the rest-of-the-world is effectively more risk-averse than the US, generating a world demand for safe (dollar) bonds. Hassan (2013) outlines a theory where the large size of the US endogenously leads to a low interest rate on dollar bonds.[4]

In addition to models of the unit of account, our paper relates to the broader literature on the role of money as both a unit of account and medium of exchange.[5] In our paper,

[3]Krugman (1984), Bruno and Shin (2015a,b), Du, Tepper and Verdelhan (2018a), Bahaj and Reis (2020), Correa, Du and Liao (2022), and Jiang, Richmond and Zhang (2022) examine different aspects of the current dollar-dominant regime in international finance.

[4]Eren and Malamud (2022) discuss how firms may optimally issue debt in dollars for exchange rate hedging motives, and Bianchi et al. (2021) study dollar liquidity premia within a search framework.

[5]For example, Kiyotaki and Wright (1989) models the endogenous rise of fiat money as a medium exchange, and in the international context Matsuyama et al. (1993) as a vehicle currency.

there is a complementarity in these two roles: financial assets are used as a liquid medium that provides settlement benefits, leading to the denomination of this medium to be chosen by issuers as the unit of account for their debt contracts, which further contributes to the liquidity of the medium of exchange in that unit of account. We model liquidity using the search framework of Duffie, Gârleanu and Pedersen (2005), and our theory features increasing returns to scale in search, as reviewed by Weill (2020) in a financial markets context.[6] In our model, the government deepening the asset pool that trades in a given unit of account will act as a spark for currency dominance by raising that medium's liquidity benefit of settlement and ignite these mutually reinforcing dynamics.

Our paper also belongs to the broader literature on the role of safe assets in the economy. Theoretical work in this area explores the macroeconomic and asset pricing implications of safe asset shortages (Holmström and Tirole 1998; Caballero et al. 2008; Caballero and Krishnamurthy 2009), and fiscal limitations in the creation of safe assets (Obstfeld, 2012). There is also an empirical literature documenting how the supply of safe impacts asset prices and quantities (Krishnamurthy and Vissing-Jorgensen 2012; Gorton et al. 2012). Our model shows that the supply and denomination of safe assets affects the determination of the dominant currency and the convenience yields of safe assets denominated in that currency. Dollar safe assets carry a convenience yield (Du et al., 2018b; Jiang et al., 2021; Du and Schreger, 2022), and dollar safe assets are held as reserve assets by central banks around the world (Ilzetzki, Reinhart and Rogoff, 2019). Our model predicts that the growing accumulation of dollar reserves by central banks around the world reflects these institutions holding the assets necessary to facilitate liquidity provision in the denomination of firms' financial obligations.

Finally, our model sheds light on the history of reserve currencies, incentives of the sovereign, and the architecture of the international monetary system. Work on the historical nature of the international monetary system includes Nurske (1944), Obstfeld, Dornbusch and McKinnon (1995), Tirole (2002), and Farhi and Maggiori (2018). Quinn and Roberds (2014b,a) and Bolt, Frost, Shin and Wierts (2023) describe the experience of the Dutch florin in the 17th and 18th centuries. Similarly, King (1972), Dickson (1967), Lindert (1969), Eichengreen (2008), and Kynaston (2015a,b) examine British pound dominance in the early 19th and early 20th centuries. Our theory supports the narrative of Eichengreen (2012) and Eichengreen et al. (2017) that throughout many historical episodes, financial development in the center country that deepened financial markets played crucial roles in supporting currency dominance. In addition, we highlight how financial market depth is complementary to investments in fiscal capacity, and we explain why entities outside the center country have historically been large issuers of debt in the reserve currency.

[6] Examples include Pagano (1989), Duffie et al. (2007), Garleanu and Pedersen (2007), Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008). Moretti (2020) studies an application to sovereign debt.

2 A Model of Liquidity and Debt Denomination

We consider a three-period ($t = t_0, t_1, t_2$) environment with two countries, indexed by $j \in \{A, B\}$. In each country, there is a mass L_j of risk-free liabilities that are government-backed, denominated in units of the local currency j, which can be traded to meet the liquidity needs of agents.

There are also entrepreneurs who run firms that issue safe debt (e.g., bills of exchange or corporate bonds), in mass F_j. These private borrowers make a choice of denominating the debt in their home currency or in the foreign currency. Firms have liquidity needs in a manner similar to Holmström and Tirole (1998). Asset trading occurs in a secondary market with endogenous trading frictions, as in Duffie et al. (2005). Finally, there is a continuum of homogeneous risk-neutral investors that buy the debt of firms and governments.

2.1 Within-Country Environment

We start by developing the model within a given country, where we consider the choice to issue debt of private-sector borrowers. In Section 2.2, we turn to the full case where firms may choose the currency in which to issue debt and characterize the general equilibrium.

2.1.1 Debt Issuance at t_0

There is a mass of entrepreneurs F_j in each country j. Each entrepreneur owns a firm that can issue debt to invest in a project at t_0. The project will generate profits of one at a stochastic time, either t_1 or t_2. The investment has a cost β^2, which is incurred at t_0. At t_0, the entrepreneur can raise funds for the investment by selling debt with face value of one maturing at t_2, which will be repaid using the future profits. As will become clear, the model is set up so that borrowing and investment is always profitable for the entrepreneur.

The preference of a given entrepreneur i is to maximize:

$$u_{i,j}^F = c_0 + \beta c_1 + \beta^2 c_2, \quad c_t \geq 0, \quad \beta < 1, \tag{1}$$

where c_t is consumption of the good in the country.

At t_0, the government also issues a quantity L_j of government-backed securities. We assume that both government and private bonds are safe (i.e., there is no default risk, as we elaborate on in Section 2.3), and that they have the same liquidity properties—hence, they are priced the same in equilibrium. We consider breaking this symmetry later in the analysis. We also consider the case of risky bonds later in the analysis. For now, as the bonds are identical they have the same endogenous price $P_{0,j}$. Further, the bonds are real, as they are each claims to one unit of the consumption good.

There is a mass I_j of investors who have sufficiently large endowments to purchase bonds

issued by the government and firms at t_0. The investors are risk neutral with preferences

$$u_{i,j}^I = c_0 + \beta c_1 + \beta^2 c_2, \qquad c_t \geq 0. \tag{2}$$

Each investor potentially owns one bond, and bonds are indivisible. The total mass of bonds is $L_j + F_j$. Define the total mass of bondholders to be

$$m_{I,j} = L_j + F_j \leq I_j, \tag{3}$$

where the last inequality is a restriction on parameters. That is, we assume there are enough investors to purchase all of the bonds at t_0.

2.1.2 Money Market Settlement With Search Frictions

The liquidity need in the model arises if the entrepreneur's profits arrive early at t_1 while his debt is due at t_2. In this case, his debt and profits streams are mismatched in time. The probability of receiving profits early is ϕ. For much of the analysis we consider the parameterization that $\phi \geq \frac{1}{2}$, meaning that liquidity needs are sufficiently large. The mass of liquidity-demanding firms is the total number of early firms, which, by the law of large numbers, is

$$m_{F,j} = \phi F_j. \tag{4}$$

With the possibility of early revenues, it may be beneficial for firms to obtain a financial asset at t_1 because such an asset would allow them to have a savings vehicle to extinguish their t_2 debt obligations. The financial assets that firms seek are the bonds issued by the government and other firms at date t_0. We model obtaining financial assets from the money market as search and matching.

Firms with early profits may trade with investors at t_1 to obtain the financial asset (i.e., a bond) for settlement at t_2. There are gains from trade in a meeting. We assume that if the firm does not trade with the investor, then it keeps its profits until t_2 and uses the profits to settle its debt. The effective return on keeping its profits is therefore zero. However, since investors discount the future at rate $\beta < 1$, an investor who owns a bond is willing to sell the bond as long as he receives at least a quantity β of goods that he consumes at t_1. The gains from trade in a match between investor and firm is therefore $1 - \beta$. We assume the firm receives a fraction η of this surplus and the investor keeps the remaining $1 - \eta$ share.

We now describe the search market at t_1. We posit a matching function such that the number of meetings between liquidity demanders (firms) and liquidity suppliers (date t_0 investors) is

$$n_j = \lambda_j m_{F,j}^\theta m_{I,j}^\theta, \qquad \frac{1}{2} < \theta \leq 1. \tag{5}$$

Here $\lambda_j > 0$ captures the overall degree of *liquidity* of the money market. In the continuous-

time asset trading model of Duffie et al. (2005, henceforth DGP), λ_j corresponds to the Poisson probability that a given agent (say, a firm) will meet another agent (say, an investor). Given Poisson meeting rates, $\theta = 1$ so that the total number of matches is proportional to the masses of both firms and investors (Duffie et al., 2018).

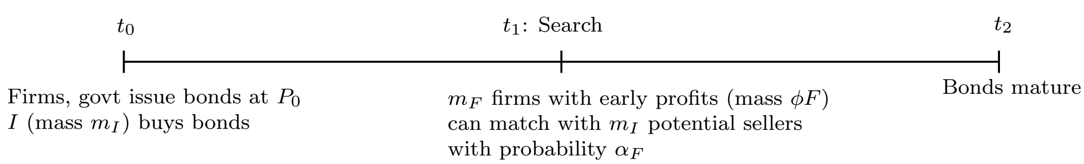
The key property of this matching function is increasing returns to scale, corresponding to $\theta > \frac{1}{2}$. If the masses of both firms and bond-holding investors double, the number of matches more than doubles. Thus the search model embeds a thick-market liquidity externality as in Diamond (1982). This liquidity externality is at the heart of our mechanism for driving dominance. We will contrast it with other forms of increasing returns that plausibly also drive dominance.

Given the matching function, the endogenous two-sided meeting probabilities are:

$$\underbrace{\alpha_{F,j} = \frac{n_j}{m_{F,j}} = \lambda_j m_{F,j}^{\theta-1} m_{I,j}^{\theta},}_{P(\text{Firm finds a bond seller})} \qquad \underbrace{\alpha_{I,j} = \frac{n_j}{m_{I,j}} = \lambda_j m_{F,j}^{\theta} m_{I,j}^{\theta-1}}_{P(\text{Bond seller finds a firm})}. \tag{6}$$

Here $\alpha_{F,j}$ is the probability of a firm meeting a bond seller (date t_0 investor in bonds) at time t_1, and $\alpha_{I,j}$ is the probability that the bond seller meets a firm. The key economic force that the modeling captures is that the trade to obtain assets for settlement is frictional, and that a greater outstanding quantity of bonds makes obtaining this liquidity easier (higher $\alpha_{F,j}$). Figure 1 provides a timeline of the debt market.

Figure 1: **Timeline of debt issuance and demand for settlement**



Notes: This figure presents a schematic representation of timing in the model.

The date t_1 liquidity market as described is an over-the-counter (OTC) bond market as in Duffie et al. (2005), where firms trade goods with investors for their one-period bonds. However, we do not take a stand on the market structure of this trade, and it is likely that this structure has varied across history, as we describe in Section 2.3.

2.1.3 Firm Issuance Decision and Asset Market Equilibrium

We formally present the entrepreneur's decision problem. The entrepreneur makes an issuance decision at date t_0. Denote D_i as an indicator function that takes the value one if the firm issues debt to invest and zero if the firm does not. The firm decides at date t_1 to trade for a bond or not. Denote T_i as an indicator function that reflects the decision to trade. Then the

entrepreneur's problem is:

$$\max_{D_i, T_i} \; E[c_0 + \beta c_1 + \beta^2 c_2]. \tag{7}$$

The structure we have posited imposes several constraints on this problem. Consumption at date t_0 is $c_0 = D_i(P_{0,j} - \beta^2)$. Consumption at date t_1 is $c_1 = D_i T_i \eta (1-\beta)$ if the profits arrive early and the firm matches with a counterparty in the asset market, and zero otherwise. Consumption at date t_2 is always zero in equilibrium, $c_2 = 0$. Then, as long as $P_{0,j} \geq \beta^2$ and $1 - \beta > 0$, the solution is to set $D_i = 1$ and $T_i = 1$.

We solve for $P_{0,j}$ backwards. Consider the market at date t_1 first. If a match occurs, the total surplus is $1 - \beta$, of which a bond seller obtains $(1-\eta)(1-\beta)$. We assume that the date t_0 bond market is Walrasian. Each investor can bid for exactly one bond at date t_0. If an investor purchases a bond at t_0, the investor either resells the bond at date t_1 to earn $\beta + (1-\eta)(1-\beta)$, or the investor holds the bond to maturity. Thus the investor's valuation of the bond at t_0 is:

$$P_{0,j} = \underbrace{\alpha_{I,j}\beta\left[\beta + (1-\eta)(1-\beta)\right]}_{P(\text{Matched}) \times \text{PV of Sales Price}} + \underbrace{(1-\alpha_{I,j})\beta^2}_{P(\text{Not Matched}) \times \text{PV of } 1}, \tag{8}$$

or rewriting:

$$P_{0,j} = \beta^2 + \alpha_{I,j}\beta(1-\eta)(1-\beta). \tag{9}$$

Since $1 - \eta > 0$, we have that $P_{0,j} > \beta^2$, so that $D_i = 1$ in the firm's issuance problem.

We define the wedge $P_{0,j} - \beta^2$ as a *convenience yield* on bonds issued at t_0. Consider the pricing of a completely illiquid bond, which in our model is one for which $\alpha_{I,j} = 0$. This bond will be priced at β^2. The government and private firm bonds in our model are priced at $P_{0,j} > \beta^2$ because they offer settlement liquidity to firms at date t_1. The convenience yield increases in the match probability ($\alpha_{I,j}$) and the surplus gained from the match ($(1-\eta)(1-\beta)$).

Finally, consider the entrepreneur's expected utility from bond issuance at date t_0 at an endogenous price $P_{0,j}$:

$$u^F_{i,j} = \underbrace{P_{0,j} - \beta^2}_{\text{Convenience yield}} + \underbrace{\beta\,\phi\,\alpha_{F,j} \times \eta(1-\beta)}_{\text{Benefit of liquidity at } t_1}. \tag{10}$$

The first two terms in this objective reflects the benefit from selling bonds at a high price at t_0 minus the cost of investment, which is the convenience yield. The second term reflects the benefit of settlement liquidity at t_1. The firm is early with probability ϕ and obtains the needed liquidity with probability $\alpha_{F,j}$. The share of the surplus in the trade that the firm receives is $\eta(1-\beta)$, and the firm discounts the future at β.

Given equilibrium bond prices, we can rewrite (10) as:

$$u^F_{i,j} = \lambda_j \beta(1-\beta)\left(m_{F,j} m_{I,j}\right)^{\theta-1}\left[(1-\eta)m_{F,j} + \phi\eta\, m_{I,j}\right]. \tag{11}$$

The two additive terms in this expression reflect the two ways in which firms benefit from money market liquidity: the first term reflects the benefit of capturing convenience yields on the firms' initial issuance, which is increasing in $m_{F,j}$, the mass of firms demanding settlement liquidity, and in $1-\eta$, the surplus share accruing to the owner of the bond if a trade happens. The second term reflects the benefit from a high probability of being able to find a match in the date t_1 money market, which is increasing in the mass of available bonds $m_{I,j}$, in the surplus share η going to the firm needing settlement, and in ϕ, the probability that the firm needs settlement.

In equation (11), both benefits accrue to the same issuers because firms are homogeneous. In practice, certain issuers can be best thought of as pure liquidity providers who harvest the convenience yield but do not have large transaction needs. On the other hand, other issuers may not be able to capture convenience yields because their debt does not offer sufficient liquidity benefits (e.g., because of default risk), but may still value settlement liquidity—and hence may be best thought of as pure liquidity demanders. In Section 5.3, we consider an extension in which we separate these two roles in the cross-section of firms.

From now on, we set $\eta = \frac{1}{2}$, as we do not explicitly model the bargaining process and η plays no part in the analysis. A firm's expected utility from debt issuance is therefore:

$$u^F_{i,j} = \frac{1}{2}\lambda_j \beta(1-\beta)\left(m_{F,j} m_{I,j}\right)^{\theta-1}\left[m_{F,j} + \phi\, m_{I,j}\right]. \tag{12}$$

2.2 International Equilibrium Conditions

We next describe the international equilibrium. The two countries, $j = A, B$, have fundamentals $\{\lambda_j, L_j, F_j\}$. The consumption goods are differentiated across the two countries, and they can be exchanged at a real exchange rate, in a process we describe below. Firms earn revenues in domestic currency (i.e., revenues are paid in the domestic consumption good) and choose the denomination of bonds, either domestic or foreign. We assume that the government only issues bonds in its domestic currency unit. While in this baseline environment we only consider real bonds, Section 4.3 extends the model to allow for nominal assets, and it considers the choice between real and nominal denomination.

Currency mismatch costs. We let the real exchange rate between the A and B consumption goods be X_t. The exchange rate process is exogenous to the model, and for simplicity, we normalize $X_0 = X_1 = 1$. At date t_2, the exchange rate is stochastic and takes one of two values $X_2 \in \{1+\gamma, 1-\gamma\}$ with equal probability, where $\gamma > 0$.

Suppose that a firm with revenues in B goods chooses to issue debt denominated in units of the A currency. The potential benefit of doing so arises if country A's debt market is relatively more liquid than B's, as we explain below. If the firm's revenues realize early, we have assumed that the exchange rate is one so that the firm is able to convert its revenues

from currency B into A with no exchange rate risk, and can then use the proceeds to repay its debt at date t_2. However, suppose the firm is instead late with its revenues of one. At t_2, the exchange rate realizes, with a depreciation of currency B to $1-\gamma$ being a bad state for the firm: in currency A units, the firm now has revenues of $1-\gamma$ and a debt obligation of one. We assume that in this bad state, the firm i can pay a disutility cost of $\kappa_i\gamma > 0$ to make up for the lost revenue.[7] We assume that there is heterogeneity in the cost κ_i across firms, and we define

$$K_i \equiv \frac{1}{2}(1-\phi)\gamma\kappa_i, \tag{13}$$

as the expected private cost of taking on currency mismatch. We assume that K_i is distributed on $[\underline{K}, \infty)$ with cumulative distribution function $H(K_i)$, and corresponding density $h(K_i) = H'(K_i)$. This density is identical in the two countries.

Finally, there is another case to consider for completeness: suppose a firm in B issues debt in B units, but converts its early revenues to A goods at t_1 to take advantage of potential liquidity benefits of the A debt market. Our assumption that $\phi \geq \frac{1}{2}$ renders this strategy dominated by that of issuing debt in A and using the liquidity of the A market. If the firm follows this latter strategy, it has one unit of the A good at date t_1, which it carries forward to date t_2 either via storage or via the liquidity of the A bond. In either case, the firm has one A good at time t_2 which it then must convert back to B goods at exchange rate X_2. The bad state of the world now is if the A currency depreciates, in which case the firm has revenues of $1-\gamma$ B goods and a debt obligation of one. Again, we assume that the firm makes up for the lost revenue by paying a disutility cost of $\kappa_i\gamma > 0$. The expected cost of this strategy is then $\frac{1}{2}\phi\gamma\kappa_i$. For $\phi \geq \frac{1}{2}$, this cost is weakly larger than K_i.[8] Our assumption that $\phi \geq \frac{1}{2}$ rules out this strategy, so that we only need to consider foreign currency denomination choices, which are the focus of our paper.

Equilibrium conditions. We let $\mathcal{M} = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$ be the set of buyer and seller masses. We can compute expected utility for the entrepreneurs in the two countries and for each of the possible denomination choices. These four expressions are as follows:

1. *Expected utility of entrepreneur in country B issuing in foreign currency (A)*:

$$U_{B\to A}(\mathcal{M}, K_i) \equiv \frac{\beta(1-\beta)}{2}\left[\lambda_A \left(m_{F,A}m_{I,A}\right)^{\theta-1}\left[m_{F,A} + \phi\, m_{I,A}\right] - K_i\right]. \tag{14}$$

[7]The disutility cost is a modeling device that ensures that firms reckon some cost due to currency mismatch, while also ensuring that the firm does not default so that the bond is riskless (consistent with our earlier assumption that private bonds are perfect substitutes for government-backed liquidity).

[8]The modeling of expected currency mismatch cost with $\kappa_i > 0$ and $\phi > \frac{1}{2}$ is one way of modeling the cost side of the denomination decision. Here is another: suppose that firms in B face a fixed cost F_i of accessing the A market, either to issue bonds or to store liquidity. Moreoever, suppose that $\kappa_i = 0$. In this case, the strategy of a B firm to issue bonds and gain the convenience yield and liquidity benefits of the A currency dominates the strategy of issuing bonds in B and using the date 1 liquidity market. The cost is the same of either strategy (F_i) while the benefit in the former is both the convenience yield and liquidity benefit, while it is only the liquidity benefit in the latter. With $K_i = F_i$, the analysis is the same as that of currency mismatch.

2. *Expected utility of entrepreneur in country B issuing in home currency (B)*:

$$U_{B\to B}(\mathcal{M}) \equiv \frac{\beta(1-\beta)}{2}\lambda_B \left(m_{F,B}m_{I,B}\right)^{\theta-1}\left[m_{F,B}+\phi\, m_{I,B}\right]. \tag{15}$$

3. *Expected utility of entrepreneur in country A issuing in foreign currency (B)*:

$$U_{A\to B}(\mathcal{M},K_i) \equiv \frac{\beta(1-\beta)}{2}\left[\lambda_B \left(m_{F,B}m_{I,B}\right)^{\theta-1}\left[m_{F,B}+\phi\, m_{I,B}\right]-K_i\right]. \tag{16}$$

4. *Expected utility of entrepreneur in country A issuing in home currency (A)*:

$$U_{A\to A}(\mathcal{M}) \equiv \frac{\beta(1-\beta)}{2}\lambda_A \left(m_{F,A}m_{I,A}\right)^{\theta-1}\left[m_{F,A}+\phi\, m_{I,A}\right]. \tag{17}$$

We index denomination choice by $\psi_{i,j}$, where $\psi_{i,j}=1$ if firm i in country j issues in foreign currency, and $\psi_{i,j}=0$ otherwise. Each firm chooses its debt denomination optimally by comparing the expected utility functions given above:

$$\psi_{i,j} = \begin{cases} 1 & \text{if} \quad U_{j\to j'}(\mathcal{M},K_i) > U_{j\to j}(\mathcal{M}), \\ 0 & \text{if} \quad U_{j\to j'}(\mathcal{M},K_i) \leq U_{j\to j}(\mathcal{M}). \end{cases} \tag{18}$$

We then have three results. First, since $U_{j\to j'}$ is monotonically decreasing in K_i, we obtain:

Lemma 1. *Consider firms $\hat{i}$ and i in country j, where $K_i < K_{\hat{i}}$. If it is optimal for firm $\hat{i}$ to issue in foreign currency $j' \neq j$, then it is also optimal for firm i to issue in foreign currency j'.*

Next, notice that the expressions for firm utility in country A have the same terms as the expressions for firms in country B. As a result:

Lemma 2. *Suppose that there is a positive mass of firms in j that find it optimal to issue in currency j'. Then, no firms in j' will issue in currency j.*

In other words, if some firms in B choose to pay a cost to issue in country A, then a firm in A for which there is no cost to issue in A will choose to only issue in A, and hence no firm from A issues in B. Together, these two lemmas imply that optimal firm denomination choices must have a threshold structure, which we formalize as follows.

Lemma 3. *A necessary condition for a collection of firm denominations choices $\psi_{i,j}$ to be consistent with firm optimality is that it must take the following threshold form:*

$$\psi_{i,j'} = \begin{cases} 1 & \text{if} \quad K_i < \bar{K}, \\ 0 & \text{if} \quad K_i \geq \bar{K}, \end{cases} \qquad \psi_{i,j} = 0. \tag{19}$$

In the lemma above, $\bar{K}$ corresponds to the threshold cost below which firms choose to issue in foreign currency: it is a scalar that provides a sufficient statistic summarizing the entirety of the set of all firms' denomination choices. A corollary is that the masses $\mathcal{M}$ can themselves be represented as functions of the threshold cost $\bar{K}$: $\mathcal{M} = \mathcal{M}(\bar{K})$. Further, we introduce the following notation for the expected utilities of the threshold firm (for which $K_i = \bar{K}$):

$$\bar{U}_{j'\to j}(\bar{K}) \equiv U_{j'\to j}(\mathcal{M}(\bar{K}), \bar{K}), \qquad \bar{U}_{j'\to j'}(\bar{K}) \equiv U_{j'\to j'}(\mathcal{M}(\bar{K})). \tag{20}$$

We denote the equilibrium value of $\bar{K}$ as $\hat{K}$. This is the cost which sets the marginal firm indifferent between the two currencies, equalizing the two expressions in equation (20).[9] Given the threshold structure for firms strategies, we can formally define equilibria.

Definition 1. *An equilibrium is a collection of an endogenous threshold $\hat{K}$, exogenous parameters $\Theta = (F_A, F_B, L_A, L_B, \lambda_A, \lambda_B, \phi, \theta)$, an exogenous firm size distribution $H(K)$, and endogenous masses $\mathcal{M} = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$, satisfying:*

1. *(Market clearing) Given $\hat{K}$, the masses $\mathcal{M}$ satisfy:*

$$m_{F,j} = \phi \left[F_j + H(\hat{K}) F_{j'} \right], \qquad m_{F,j'} = \phi \left(1 - H(\hat{K}) \right) F_{j'}, \tag{21}$$

and,

$$m_{I,j} = L_j + F_j + H(\hat{K}) F_{j'}, \qquad m_{I,j'} = L_{j'} + \left(1 - H(\hat{K}) \right) F_{j'}. \tag{22}$$

2. *(Firm optimality) Given the masses $\mathcal{M}$, the threshold is optimal: firms in j' with $K_i < \hat{K}$ find it optimal to issue in currency j, while all other firms optimally issue in their own currency, so that*

$$\begin{cases} \bar{U}_{j'\to j}(\hat{K}) = \bar{U}_{j'\to j'}(\hat{K}) & \text{for} \quad \hat{K} > \underline{K}, \\ \bar{U}_{j'\to j}(\hat{K}) \leq \bar{U}_{j'\to j'}(\hat{K}) & \text{for} \quad \hat{K} = \underline{K}. \end{cases} \tag{23}$$

Further, throughout the analysis, we will examine the stability properties of the equilibria specified in the above definition, whenever the model features multiple equilibria. We formally specify our notion of stability using the following criterion.

Definition 2. *Consider an underlying dynamical system through which an out-of-equilibrium system converges to equilibrium (a tatonnement process) of the form $\partial_\tau \hat{K} = \delta \left[U_j(\hat{K}) - U_{j'}(\hat{K}) \right]$ for $\delta > 0$ and where τ indexes a mass of continuous sub-periods within time t_0. An equilibrium featuring the endogenous threshold $\hat{K}$ is said to be stable if it fulfills the following condition:*

1. *(Equilibrium stability) There exists an $\varepsilon > 0$ such that any trajectory beginning in the neighborhood $[\hat{K} - \varepsilon, \hat{K} + \varepsilon]$ converges to $\hat{K}$.*

[9] Alternatively, by complementary slackness, equilibria at the lower boundary ($\hat{K} = \underline{K}$) are also valid if $\bar{U}_{j'\to j}(\underline{K}) \leq \bar{U}_{j'\to j'}(\underline{K})$.

2.3 Discussion of Modeling Choices

We further explain some of the modeling choices that we have made in this section.

1. *Rollover risk vs. saving:* The liquidity trade in our model at t_1 is firm "saving", but this is more for simplicity than it is substantive. The key component is that the firm trades its goods for a one-period bond. Consider a variant of the model in which there is a date t_3, and the firm's liquidity need arises from rollover risk. The timing mismatch in that problem is that with some probability, the firm will receive the goods at t_3 rather than t_2 while the debt is due at t_2. The firm will then want to trade its future revenues for bonds at t_1. If there are more bonds available, this trade will be less frictional and it will be less costly to rollover the debt. Again, the key economics our model captures is in linking liquidity with bond supply.

2. *Is search and liquidity a concern in money markets?* Our model links corporate financing decisions to money market illiquidity concerns. Our analysis turns on the relative liquidity of the money markets in different currencies rather than the absolute liquidity of any one market. Empirically, even in high-volume money markets such as the US dollar repo market, search models have been shown to capture price and quantity patterns well (Vayanos and Weill, 2008). At the macro level, during a period of global financial volatility, the dollar money market remains more liquid than the markets of many emerging and even advanced economies. In our model, these considerations drive financing decisions.

3. *Interpretation of L.* The mass of government-backed liabilities L should be interpreted as the supply of money market instruments that depends on the government's commitment to maintain their value. Throughout history, government commitment to a pool of liquid assets has taken various forms: through physical holdings of precious metals such as in a metallic standard for a currency or purely fiscal commitment as is more common in the post-WWII era. We incorporate government default risk in the model in Section 5.5.

4. *Institutional structure of the money market at t_1:* The asset trade between investors and firms, in which investors that have bought safe debt at t_0 provide liquidity at t_1, often takes the form of financial intermediation. For example, these labels apply in thinking about a banking arrangement or modern money market and repo in which the "liquidity suppliers" (m_I in the model) are banks that own the bonds as an asset and issue deposits to "liquidity demanding" firms (m_F) at t_1, which the firms then use to settle their t_2 debt. Alternatively, financial firms use Treasury securities as collateral in repo arrangements to obtain reserves that can then be lent to others as settlement instruments.

   In the global dollar market, liquidity supply is provided by foreign central banks and global banks. These entities hold safe dollars as foreign reserves, obtained at t_0 in the

form of Treasury securities, which they provide to firms at t_1. In this way, reserve holdings would tilt towards the dollar even without asymmetry in investor preferences for dollar assets.

3 Currency Dominance and Denomination Incentives

Having specified the model environment and derived its equilibrium conditions in Section 2, we now turn to analyzing the properties of the resulting equilibria and examining the underlying economic forces. Multiple equilibria naturally emerge as a consequence of the liquidity compementarity, and we outline how asymmetries in country fundamentals that generate asymmetries in financial market liquidity favor equilibria that feature currency dominance.

3.1 Equilibrium Characterization

The following proposition characterizes the configuration of model equilibria, allowing for potentially asymmetric country fundamentals, heterogeneous positive costs $K_i \in [\underline{K}, \infty)$, and a degree of returns scale $\theta \in \left(\frac{1}{2}, 1\right]$.

Proposition 1 (**Equilibrium characterization**). *The model can generate three classes of stable equilibria:*

1. *Interior equilibria featuring issuance from country B firms intro currency A (class BA).*
2. *Interior equilibria featuring issuance from country A firms into currency B (class AB).*
3. *No foreign denomination equilibria, in which no firm switches to issues in the foreign currency.*

Depending on the parameter vector Θ, these different classes of equilibria can all be present at the same time: hence, the model features equilibrium multiplicity. Alternatively, for particular parameter configurations, stable equilibria of only one class or of only two classes can be present.

Proof. See Appendix Section A.2. □

To understand the economic forces shaping this equilibrium configuration, we now provide a discussion and graphical analysis where, for ease of illustration, we parameterize $\theta = 1$. We also set $H(K)$ as a Pareto distribution, so that the cumulative distribution function takes the form $H(K) = 1 - \left(\frac{\underline{K}}{K}\right)^{\alpha}$ where $\alpha > 0$ is a shape parameter. This distribution features several properties that are salient in the cross-section of firms. In particular, it captures the notion that most debt issuance is done by a tail of very large firms with low per-unit costs K_i.[10]

[10] The size distribution of firms is fat-tailed and well-described by a Pareto form (Gabaix 2011, Chaney 2018). In addition, we expect foreign currency issuance fixed costs (Maggiori et al. 2020) to give rise to a negative correlation between costs paid per unit of debt and firm size.

These large firms will be the first to sort endogenously into foreign currency issuance, and therefore small increases in the threshold $\bar{K}$ in the neighborhood of the lower boundary $\underline{K}$ will give rise to disproportionately large movements in the masses $\mathcal{M}$, as compared to similar increases in $\bar{K}$ at higher levels of foreign denomination.

In class BA equilibria, firms in country B play the threshold strategy described in Lemma 3, and all firms in country A issue in home currency. The marginal firm in country B (one for which $K_i = \bar{K}$) is indifferent between issuing in the two currencies. Therefore an interior equilibrium threshold $\hat{K}$ satisfies the following indifference condition, which is a specialization of equation (23):[11]

$$\underbrace{\lambda_A \left[m_{F,A} + \phi\, m_{I,A}\right] - \hat{K}}_{\bar{U}_{B\to A}\text{: Utility from issuing in foreign currency}} = \underbrace{\lambda_B \left[m_{F,B} + \phi\, m_{I,B}\right].}_{\bar{U}_{B\to B}\text{: Utility from issuing in home currency}} \tag{24}$$

The masses of liquidity demanders (buyers) in the two currencies are

$$m_{F,A} = \phi \left[F_A + H(\hat{K}) F_B\right], \qquad m_{F,B} = \phi \left[1 - H(\hat{K})\right] F_B, \tag{25}$$

while the masses of liquidity suppliers (sellers) are

$$m_{I,A} = L_A + F_A + H(\hat{K}) F_B, \qquad m_{I,B} = L_B + \left[1 - H(\hat{K})\right] F_B. \tag{26}$$

Figure 2a plots the curves $\bar{U}_{B\to A}$, and $\bar{U}_{B\to B}$ as functions of the threshold cost $\bar{K}$, keeping the country fundamentals (λ_j, L_j, F_j) symmetric. These curves capture the expected utility of the marginal firm $(K_i = \bar{K})$ from issuing in foreign currency or home currency. The shapes of these two curves reflect the economic forces at work. The curve $\bar{U}_{B\to B}$ is monotonically decreasing since higher values of $\bar{K}$ correspond to higher issuance into currency A, which reduces the thickness of currency B markets, lowering the utility of home currency issuance. Conversely, higher issuance raises the expected utility of foreign currency issuance, which is a force pushing $\bar{U}_{B\to A}$ higher. The curve $\bar{U}_{B\to A}$ is however also subject to a second force, since as $\bar{K}$ increases, the identity of the marginal firm changes: it is now a firm with higher foreign issuance cost K_i. This gives rise to a linearly decreasing component of $\bar{U}_{B\to A}$. Given the Pareto distribution of costs, there is a diminishing marginal impact of foreign denomination from increasing the threshold cost $\bar{K}$, which mediates the relative strength of the forces impacting $\bar{U}_{B\to A}$, giving rise to its non-monotonic (concave) shape.

The model features three equilibria of class BA in this example. Two equilibrium points (labeled 1 and 2, and occurring at $\hat{K}_1$ and $\hat{K}_2$, respectively) lie at the intersections of the two curves $\bar{U}_{B\to A}$ and $\bar{U}_{B\to B}$. At these intersections, the expected utility from issuing in A and B is

[11] For ease of exposition, here we do not separately write out the condition for equilibria at the left boundary $\hat{K} = \underline{K}$, although these are of course valid if the complementary slackness condition in the second row of equation (23) is satisfied.

equalized for the marginal firm with threshold cost $\bar{K} = \hat{K}$. The interior equilibrium featuring low foreign denomination (point 1) is unstable while the high foreign denomination equilibrium (point 2), featuring currency dominance, is stable. A further equilibrium point (labeled 0) is at $\bar{K} = \underline{K}$. In this equilibrium, no firms issue in foreign currency: this equilibrium is now stable (relative to the $K_i = 0$ case) because of the presence of fixed costs $K_i > 0$, which make $\bar{U}_{B\to B}$ higher than $\bar{U}_{B\to A}$ in the neighborhood of $\bar{K} = \underline{K}$.

There is a second, symmetric class of equilibria as well. These are equilibria in which firms in country A issue in foreign currency, while all firms in country B remain in home currency (*class AB equilibria*). For this class of equilibria, $\bar{K}$ now characterizes the threshold strategy played by firms in country A. The analysis is analogous. Since it is now the marginal firm in country A that needs to be indifferent, the indifference condition determining an interior equilibrium threshold $\hat{K}$ in this case is:

$$\underbrace{\lambda_B \left[m_{F,B} + \phi\, m_{I,B}\right] - \hat{K}}_{\bar{U}_{A\to B}\text{: Utility from issuing in foreign currency}} \quad = \quad \underbrace{\lambda_A \left[m_{F,A} + \phi\, m_{I,A}\right]}_{\bar{U}_{A\to A}\text{: Utility from issuing in home currency}}, \tag{27}$$

and the masses $\mathcal{M}$ are obtained by performing the analogous specialization of (21) and (22).

Figure 2b provides a graphical analysis by showing the curves $\bar{U}_{A\to B}$, and $\bar{U}_{A\to A}$ as functions of the threshold $\bar{K}$. As in the previous case, there are two interior equilibria: a low foreign denomination unstable one (labeled 3) and a high foreign denomination stable one (labeled 4). The equilibrium in which no firm switches (at $\bar{K} = \underline{K}$, labeled 0) also features. This is because the equilibrium with no foreign denomination has two isomorphic representations: one as a class BA equilibrium, and one as a class AB equilibrium.[12]

We note that despite the presence of currency dominance, our model with positive foreign issuance costs rules out winner-takes-all equilibria in which only one currency survives, and the dominant equilibria remain interior. Hence in this respect, our model is consistent with the evidence in Eichengreen et al. (2017) that despite currency dominance, multiple currencies co-exist as units of denomination in equilibrium.

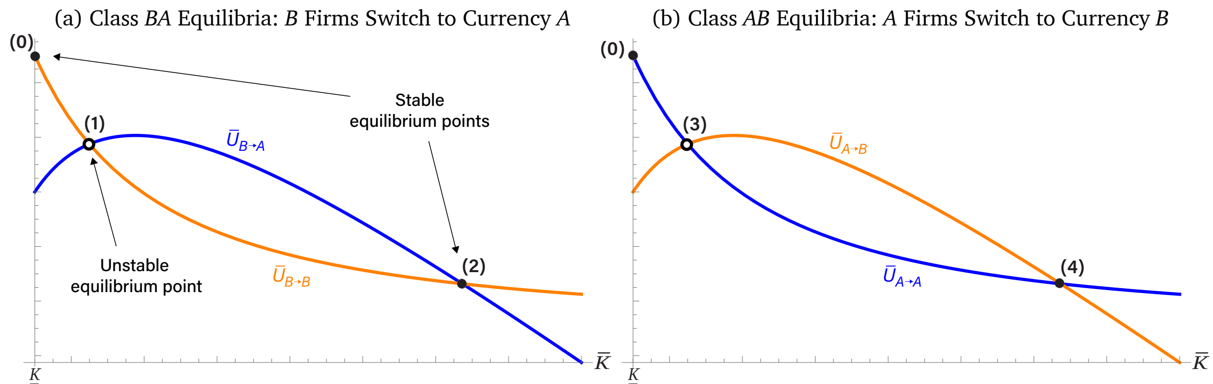
3.2 Comparative Statics Analysis and Liquidity Asymmetries

We can now consider comparative statics with respect to country fundamentals.[13] The following proposition characterizes these results.

[12] If writing down the threshold strategy from the perspective of firms in country B (class BA representation), the equilibrium with no foreign denomination corresponds to the case in which $\bar{U}_{B\to B}(\underline{K}) \geq \bar{U}_{B\to A}(\underline{K})$. If expressing the strategies from the perspective of firms in country A (class AB representation), it corresponds to the case in which $\bar{U}_{A\to A}(\underline{K}) \geq \bar{U}_{A\to B}(\underline{K})$. In either case, each condition independently implies by Lemma 2 that *all* firms in *both* countries choose to denominate in home currency.

[13] Throughout this analysis, we require that L_j and F_j denote the safely backed component of bond supply. Of course it is possible that as debt commitments rise, there is a collapse in confidence such that nominal liabilities are high but the safe portion is low, as formalized for instance in Farhi and Maggiori (2018).

Figure 2: **Characterizing equilibria in the heterogenous cost case**



Notes: Panel A plots the expected utility of a firm in country B switching to issuing in foreign currency ($\bar{U}_{B\to A}$), and of a firm in B issuing in home currency ($\bar{U}_{B\to B}$), as a function of the threshold cost $\bar{K}$. Panel B plots the expected utility of a firm in country A switching to issuing in foreign currency ($\bar{U}_{A\to B}$) or issuing in home currency ($\bar{U}_{A\to A}$), also as a function of $\bar{K}$. The currency-switching costs K_i are distributed over $[\underline{K}, \infty)$ with a Pareto CDF $H(K)$. For illustration, the graph focuses on the case $\theta = 1$ and uses symmetric country fundamentals (λ_j, L_j, F_j). There are five equilibria.

Proposition 2 (Comparative statics). *Consider equilibria of class BA (the results for class AB equilibria are symmetric). The following comparative statics results hold at an interior, stable equilibrium:*

1. $\mathbf{L_A}$: *An increase in country A's government backed liquidity supply L_A increases foreign-currency issuance by B-firms ($\frac{\partial \hat{K}}{\partial L_A} > 0$) for any $\frac{1}{2} < \theta \leq 1$. A sufficiently high increase in L_A dissolves the equilibrium with no foreign denomination, resolving equilibrium multiplicity in favor of a high foreign denomination equilibrium with currency A dominance.*

2. $\mathbf{F_A}$: *The same holds for increases in the mass F_A of firms in country A, such that $\frac{\partial \hat{K}}{\partial F_A} > 0$, under the parameter assumption $(1-\theta)\left[1 + \frac{L_A}{F_A+F_B}\right] < \theta \leq 1$. This parameter assumption is more stringent than $\frac{1}{2} < \theta \leq 1$ and hence requires a sufficiently high degree of returns to scale.*

3. $\boldsymbol{\lambda_A}$: *The same holds for increases in the overall matching intensity λ_A, so that $\frac{\partial \hat{K}}{\partial \lambda_A} > 0$ for any $\frac{1}{2} < \theta \leq 1$.*

4. $\mathbf{F_B}$: *The sign of the comparative static $\frac{\partial \hat{K}}{\partial F_B}$ is ambiguous and depends on the level of foreign denomination. For sufficiently high foreign denomination $\hat{K}$, the comparative static becomes unambiguously positive.*

Proof. See Appendix Section A.3. □

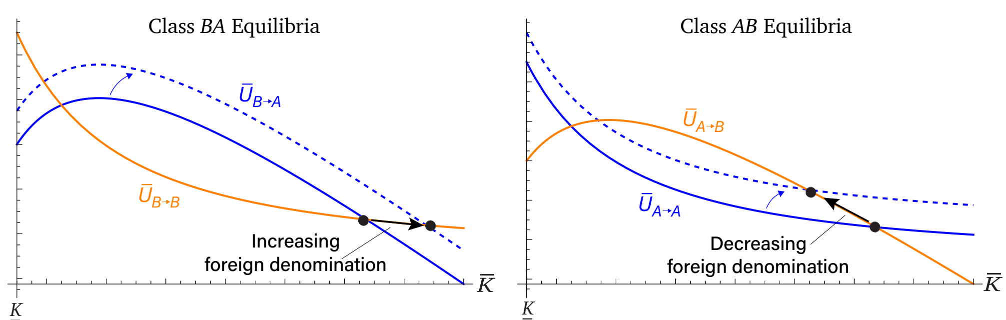
Figures 3a and 3b provide a graphical exposition of these comparative static results, again in the illustrative case with symmetric fundamentals and $\theta = 1$. Consider first an increase in government backed liquidity supply L_A. As shown in Figure 3a, this acts as an

outward shift of the blue curves $\bar{U}_{B\to A}$ and $\bar{U}_{A\to A}$: as government commitment to its liabilities grows, the mass of investors $m_{I,A}$ that are sellers in the A money markets increases, which in turn benefits issuers in currency A's market as it expands the pool of liquidity available to them at date t_1. If this shift occurs starting from the high foreign denomination class-BA equilibrium (point 2), the equilibrium threshold $\hat{K}$ shifts further to the right, which means foreign denomination is increasing, and currency A's dominance is more entrenched. If the shift occurs starting from the high foreign denomination class-AB equilibrium (point 4), $\hat{K}$ conversely shifts to the left, as the resulting asymmetry in fundamentals weakens currency B's dominance.

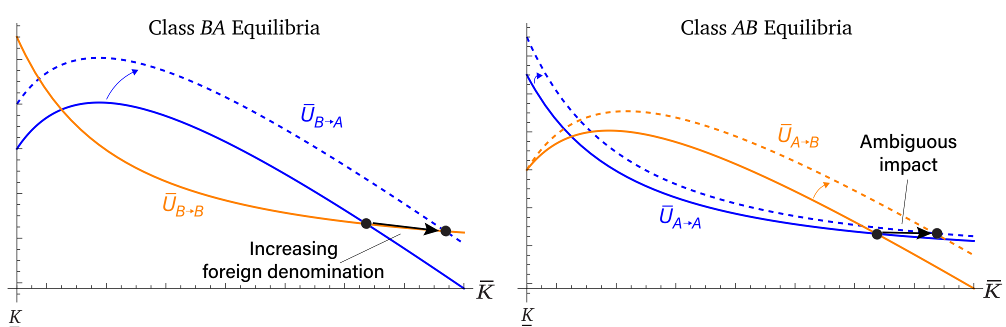
Crucially, increasing L_A sufficiently will cause a qualitative shift in the configuration of equilibria in both equilibrium classes. First, a sufficient upward shift of the curve $\bar{U}_{A\to A}$ will raise $\bar{U}_{A\to A}$ above $\bar{U}_{A\to B}$ over the entire domain $[\underline{K}, \infty)$ for class AB equilibria so that the two curves never intersect. Intuitively, this implies that sufficiently strong asymmetry between L_A and L_B dissolves all class AB equilibria in favor of class BA equilibria. Simply put, equilibria in which B is dominant cannot survive once country A achieves a sufficiently large advantage in government debt supply, all other fundamentals equal.

Figure 3: **Comparative statics**

(a) Increasing government bond supply L_A



(b) Increasing firm mass F_A



Notes: We consider introducing asymmetry in country fundamentals by increasing government bond supply L_A (Panel A) or the mass of firms F_A (Panel B).

Second, large enough increases in L_A also impact the configuration of BA equilibria themselves. This happens once the $\bar{U}_{B\to A}$ curve crosses above $\bar{U}_{B\to B}$ at $\hat{K} = \underline{K}$. Once this

threshold is crossed, the no foreign denomination and low foreign denomination equilibria (points 1 and 2) both disappear, leaving the high foreign denomination equilibrium as the sole remaining one. These results illustrate another key economic point, which we discuss further below: multipolar equilibria (with denomination dispersed among multiple competitor currencies) only survive in a world of roughly symmetric fundamentals, while sharp asymmetries among countries result in dominance.

Next, consider an increase in the mass of firms F_A in country A, the effects of which are shown in Figure 3b. Growing the size of the private sector is *not* equivalent to increasing safe government debt supply: while these have the same effect from the perspective of class BA equilibria, the same is not true for class AB equilibria. Similar to an increase in L_A, increasing F_A shifts up the curve $\bar{U}_{A\to A}$, since a share of the additional mass of A firms $(1 - H(\hat{K}))$ will continue to issue in home currency, improving the liquidity of the currency A markets. On the other hand, some of the additional mass will go towards improving the liquidity of currency B markets, in proportion to the share of A firms $(H(\hat{K}))$ that issues in foreign currency. For $\bar{K} > \underline{K}$, increasing F_A therefore tilts up the $\bar{U}_{A\to B}$ curve as well, which has the effect of *increasing*, rather than decreasing, issuance of A firms in currency B.

The net impact of increasing F_A on foreign denomination in class AB equilibria therefore depends on the relative strength of these two forces, which depends on the value of $\hat{K}$: for high value of $\hat{K}$, the second force will tend to be relatively stronger, and vice-versa. Figure 3b shows an example in which the starting value of $\hat{K}$ in the stable AB equilibrium is sufficiently high that the second force is stronger, and as a result, the increase in F_A leads to overall higher foreign denomination, unlike in the case of increases in L_A.

These different forces illustrate a fundamental difference between sovereign and private issuance in our model: if a country does not start out as a dominant currency issuer, simply growing the size of the private sector is not guaranteed to facilitate the internationalization of its currency, and in fact can be counterproductive. Increasing the stock of safe government liabilities is instead a more reliable instrument, as it will facilitate international usage of the currency regardless of the starting equilibrium conditions.

4 Evidence for the Liquidity Force

In the last four centuries of the international monetary system, the dominant currency in global finance has been the Dutch florin followed by the British pound sterling and then the US dollar. In this section, we illustrate how historical and modern experiences of dominance can only be rationalized by the liquidity force emphasized by our theory.

4.1 Liquidity Versus Economic Size Throughout History

The first prong of our argument concerns the historical relationship between economic size, financial liquidity, and currency dominance. Throughout the multiple historical experiences that we review in this section, currency dominance accrued not to the economies with the largest size or trade volumes, but rather to those which established superior financial liquidity, backed by sizable and credible government commitments. Within the language of our theory, the counterexamples are countries with large economic size F_j but small government commitment L_j.

4.1.1 Financial market liquidity as a seed for dominance

Dutch dominance. The first dominant currency to emerge in early modern history is the Dutch florin, created by the Bank of Amsterdam in 1609. Amsterdam was uniquely able to commit to creating a currency that provided much bigger settlement benefits, thereby launching the liquidity-based complementarity that we model. Its florin-denominated money market hosted international transactions for almost two centuries even though in economic size, trade volumes, and overall wealth, Amsterdam was much smaller than competitor nations such as Spain and France.

During the early 17th century when the Bank of Amsterdam was created, payments were primarily made in metallic coin. However, metallic coins were illiquid: at any given time and place that a debt needed to be settled, there was an uncertain supply of high quality coins in specific denominations because coins were frequently transported, debased, or melted down. There were nearly 1,000 gold and silver coin denominations circulating in Amsterdam at the time the florin was created, which led to large settlement frictions (Neal, 2000).

The Dutch florin, in contrast, was a pure unit of account that lived on the ledgers of the Bank of Amsterdam. As a result, the florin did not suffer any of the usage frictions of metallic coin. Amsterdam's key innovation was to pool together disparate assets, such as various kinds of circulating metallic coins and specie, to back a single, unified debt instrument defined as the florin.[14]

By the late 17th century, the Bank had innovated to grow its balance sheet through the use of a "repo" facility as well as open market operations. In the repo transaction, a trader could deposit specie or coins with the bank and receive florin in his account, plus a "receipt" to exchange that quantity of florin back into coin at a future date. These traders would pay a fee to the Bank in this transaction and would therefore receive less florin than the value of the specie, as in an overcollateralized repo transaction.[15] Receipts traded separately on an

[14]Amsterdam's liquidity pool was further augmented by other Dutch cities following Amsterdam's lead rather than competing with their own currencies. See Appendix B.1 for more details on the arrangement between Rotterdam and Amsterdam, for example.

[15]In the context of the model, a t_0 investment took the form of traders depositing their specie at the Bank to receive a florin deposit. The traders would then lend florin to a merchant at t_1 who needed florin for settlement,

open market, and anyone concerned about the florin's backing could buy a receipt in order to demand repayment of his coin.

The Bank could also create florin by directly purchasing coins at the market rate (the "agio"), as in an open market operation (Quinn and Roberds, 2019). These assets were "unencumbered" by a receipt and were not subject to being withdrawn from the Bank. Note that florin credited to an investor through such an operation were indistinguishable from florin obtained via the repo facility and were part of the Bank's homogeneous debt. The aggregate quantity of florin outstanding was the debt of the bank while the owners of the receipt (that allowed conversion from florin into coins) effectively held equity in the bank.

First, this capital structure meant that the florin was not defined in terms of a given amount of gold or silver, but rather was a financial instrument backed by disparate assets and an equity cushion. Second, this structure enabled a large and elastic supply of florin, which was a key factor that allowed it to become uniquely liquid during this period. The ease of settling obligations in the florin created a unified system of payments that could easily and immediately be accessed by anyone with an account (or a bank correspondent) at the Bank of Amsterdam.

Government backing was another key component of the appeal of the florin.[16] Amsterdam was able to create a sufficiently safe supply of florin that could meet international payment needs because of the implicit commitment of the City of Amsterdam not to appropriate the reserves of the bank.[17]

Dutch florin dominance has been discussed extensively by both contemporaries and historians. First, the prevalence of the the bill of exchange payable in Amsterdam, a financial obligation defined by its denomination in florin, is a direct measure of the florin's dominance (Van Dillen, 1934).[18] The bills on Amsterdam were much more prevalent than those of any other European city (Flandreau et al., 2009), and in fact, its success has been attributed to the depth of Amsterdam's connections with all other cities (Gillard, 2004).[19] Second, while

earning interest or a fee in this transaction. The repo technology and specie are therefore "liquidity supply" and the merchant is the "liquidity demander" at t_1.

[16] Kindleberger writes, "The convenience of a deposit at the Bank—safety of the money and assurance that one received money of satisfactory quality—meant that bank money went to a premium over currency," (Kindleberger, 1984, p. 48).

[17] A contemporary reference book described the safe backing of the florin in the following way: "The government [...], the magistry of the city of Amsterdam, in the year 1609, proposes to the merchants to take in their money, and secure it in strong vaults below their famous Stadt house, which vaults are shut with iron doors, and guarded by strong guards night and day: and besides all this, they have security on the revenues of the city, which is better than that of several crowned heads. The merchants finding the convenience, as well as the advantage of this proposal, soon brought in vast sums of money...and when their money was laid in, every one got credit in the bank books for his particular sum; and thus, by circulating payment from one to another, they could pay millions in a moment, by subscribing their names." (Stevenson, 1764, p. 215).

[18] Bills of exchange were the primary form of financial instrument during this period, used flexibly for payment, (collateralized) trade finance, or (uncollateralized) direct credit (Quinn and Roberds, 2024).

[19] "Bank of Amsterdam bills of exchange became accepted everywhere in the world and in some places were the only accepted means of payment [...] Moreover, these bills of exchange became commodities in their own right when they began to be traded on the Amsterdam Bourse. This further contributed to liquidity," (Sobel,

bills of exchange often financed real trade flows, there was no obligation for the payment for a bilateral transaction to clear in either of the two parties trading (Kindleberger, 1984, p. 41). While Amsterdam was the “clearinghouse” for world trade, the vast majority of these payments did not reflect its own trade flows, indicating that financial dominance did not need to arise out of trade dominance.

British pound sterling and US dollar dominance. The British pound sterling era was created by a similar asymmetry in government backing for a currency, this time triggered by two large wars rather than financial innovation. The first war (The Fourth Anglo-Dutch war) considerably weakened the city of Amsterdam, which led to a collapse of confidence in the Bank of Amsterdam.[20] In our model, this is equivalent to a large and sudden contraction in L_A that shifts in the U_A curve and dissolves the dominant equilibrium of foreign denomination in currency A. The second war (the Napoleonic wars) were resolved with Great Britain's victory at Waterloo, leaving the country with the largest government debt. The state capacity for taxation and credible governance made the pound-denominated debt, which was approximately 200% of GDP, safe. As in the case of Amsterdam before it, having the largest pool of safe liquid assets for settlement initiated the process of dominance, which the UK held until the mid-20th century.

As in the case of the transition from the Dutch florin to the pound, the transition from the pound to the dollar was precipitated by two wars in which there was a collapse in the size of the safe pound-denominated government-backed liquidity pool, and a switch to the deepest available pool, in US dollars. In the aftermath of the world wars, the US had the largest safe government debt pool, and for the first time since the early 19th century, a unified monetary system under the Federal Reserve System.

4.1.2 Size without liquidity did not generate dominance

Lack of government commitment. During the Dutch dominance era, Amsterdam, and the Dutch Republic more generally, was much smaller than other European states. Spain and France were the wealthiest and largest economies, and theories based on economic size would predict that one of these countries would have hosted the dominant currency. However, these countries lacked the ability to commit to a sufficiently large pool of government liabilities.

Spain, for example, was the wealthiest country with the most military power ruling over the largest global empire at the time, but its history of serial default made its debt unsafe. Therefore while the Spanish knew of the ledger technology, it was not able to commit to creating such a currency. Instead, the Spanish crown relied on physical coin for payments,

2012, p. 73).

[20]The Bank had secretly made large unsecured loans to the Dutch East India Company after the war, which had suffered large losses from the naval blockade in the war. When these loans and losses came to light, the Bank suffered a run from which it never recovered.

which partly reflected its inability to create other forms of debt obligations.

The wealth of the Spanish empire was also channeled into creating the large volumes of Spanish "dollars" (*real de ochos*) that circulated around the world. However, despite the fact that Spanish dollars were the primary metallic coin in many parts of the world, they were costly to find and use, and thereby generated low settlement benefits.[21]

France was another likely alternative to Dutch dominance with the largest population and economy, as well as colonial ventures around the world.[22] In addition, France, like Amsterdam, invested in creating a unit of account that did not depend on a specific metallic coin, known as the "livre tournois," but unlike Amsterdam, the backing and definition of this unit was unclear and often depreciated (Cipolla, 1963; Richet, 1975). In the context of the model, both Spain and France lacked sufficiently large L to sustain a liquid money market.[23]

During the British pound sterling era, France was another large country that could potentially have competed with British dominance. However, France experienced several disruptions to its supply of safe government debt during the regime changes in the late 18th century, and its loss at Waterloo reduced its subsequent ability to float a large volume. Unlike the UK, it was required to pay off its wartime debt with much higher taxes instead of new long-term issuances, which were also at higher rates, indicating a low ability to commit to backing safe liabilities (Bordo and White, 1991). While the franc itself returned to the gold standard such that the physical coins were just as safe as pounds sterling, the outstanding amounts of all other forms of franc-denominated claims were much smaller than that of the pound's, leading to large asymmetries in their relative liquidity. One illustration of pound dominance is that French firms were the largest foreign issuers in the corporate bond issuance in London, accounting for 25% on average from the mid-19th century to WWI.[24]

The transition from the pound to the dollar also provides an example of the importance of government commitment. After WWI, US government debt was much more plentiful, and dollars began to compete with pounds in global financial markets. However, in 1933, when the US devalued the dollar, the international use of the dollar shrank again even though the economy was growing, consistent with a collapse in confidence in the US government's commitment to its currency.

Finally, during the current dollar period, the emergence of the use of the euro in the late 20th century reflects a consolidation of commitment, represented by L, across multiple countries into a single denomination, which propelled the euro's rise. Similarly, the contraction

[21] Chalmers (1893) discusses the common practice of clipping Spanish coins with high quality coin quickly going out of circulation.

[22] At the beginning of the Dutch dominance period in 1600, France's population was approximately 15 times larger than that of the Dutch Republic, 3-4 times larger than of England, and 2-3 times larger than of Spain. Economically, it was 5-7 times larger than the Netherlands and 2-3 times larger than Spain and England (Sobel, 2012, p. 56).

[23] Appendix Section B.1.2 further relates the importance of the liquidity force in our model to the earlier historical experience of the city states of the Italian Renaissance.

[24] Authors' calculations based on bond data from the *Investor's Monthly Manual*.

of the euro's global usage after 2008 matches a decrease in L, reflecting new information that government debt from countries like Spain, Italy, Portugal, and Greece did not contribute to the commitment backing the euro.

Lack of financial market institutions. During the British pound sterling era, another potential competitor in terms of economic size was the United States. In the aftermath of the Civil War, the US quickly returned to the gold standard and overtook the UK in GDP. By the turn of the 20th century, the US overtook the entire British empire as the largest economy in the world, with similar volumes of trade (see Figure B.2 in Appendix Section B.3). However, throughout this period, the US did not have a central bank that could create a unified money market, and the US dollar market was fragmented regionally. Therefore, despite its large F throughout the 19th and 20th centuries, without the depth that unification from the Federal Reserve System offered, private firms in the US also continued to issue debt that traded in pounds in London. Proposition 2 indicates that firms' issuance deepened the dominance of the pound, rather than raised competition for the pound.

4.2 The Role of Trade

It has been clear both historically and today that trade and finance are intricately connected. Historically, bills of exchange were issued first and foremost to finance international trade transactions and today, trade finance continues to be an important part of the global credit market. This section clarifies how trade intensity interacts with the benefits of financial market depth that we highlight in our theory.

Trade invoicing in the model. In our baseline model, firms only receive revenues in their domestic currency. However, we can consider an extension where firms receive a portion of their revenue stream denominated in foreign currency, as in the case of traded goods revenues that are invoiced in foreign currency. For a given firm, this will reduce K_i because the receipt of some profits in the foreign currency reduces the amount of currency mismatch that a firm faces if it issues dollar-denominated debt, as in our Appendix Section A.1 modeling of K_i.

A reduction in K_i for all i is a leftward shift of $H(K)$, raising the utility of issuing abroad. Rewriting the equilibrium condition in equation (24) in terms of firm masses, considering the $\theta = 1$ case for ease of illustration, the equilibrium condition is:[25]

$$\underbrace{\lambda_A \phi \left[2F_A + L_A + 2F_B H(\hat{K})\right] - \hat{K}}_{\bar{U}_{B\to A}} = \underbrace{\lambda_B \phi \left[L_B + 2F_B(1 - H(\hat{K}))\right]}_{\bar{U}_{B\to B}} \tag{28}$$

Consider the case where there is an increase in the number of B firms receiving revenues in

[25] This argument generalizes straightforwardly to the general θ case, in which the results are analogous.

foreign currency. The cost distribution shifts left from $H_0(K)$ to $H_1(K)$ where H_0 f.o.s.d. H_1. This shift increases $H(\hat{K})$ and raises $U_{B\to A}$ while reducing $U_{B\to B}$ for the marginal firm at $\bar{K} = \hat{K}_0$. This benefit of foreign denomination will lead B firms to issue abroad until a new equilibrium is reached that equates the two sides, which will occur at $\hat{K}_1 > \hat{K}_0$. Therefore, invoicing in foreign currency leads to more financial dominance.

We can also consider how the denomination of trade invoicing might endogenously emerge in our model as a consequence of the liquidity forces discussed.[26] Although this steps outside our formal analysis, if firms were also allowed to choose their currency of invoicing, dominant currency invoicing would emerge *because* of the foreign exchange exposure firms have in their financial obligations. In that case, firms would choose to invoice in the foreign currency in order to reduce K_i, and the invoicing dominance is a byproduct of our equilibrium with financial dominance.[27]

Empirical implications. History provides examples where volumes of trade invoicing was unlikely to have been the seed of currency dominance, as in the case of economic size. In the 17th and 18th centuries during the period of Dutch florin dominance, the value of Spanish trade was several factors larger than that of Dutch trade. The pound sterling also gained prominence in the 1820s immediately after Waterloo, which pre-dates its mid-century surge in world trade and expansion of the British Empire.[28] Rather, the dating of dominance is more clearly linked to investments in financial technologies by the state and private sector that enhanced liquidity. In Section 5.1, we discuss the financial technologies that emerged in London in early 1800s.

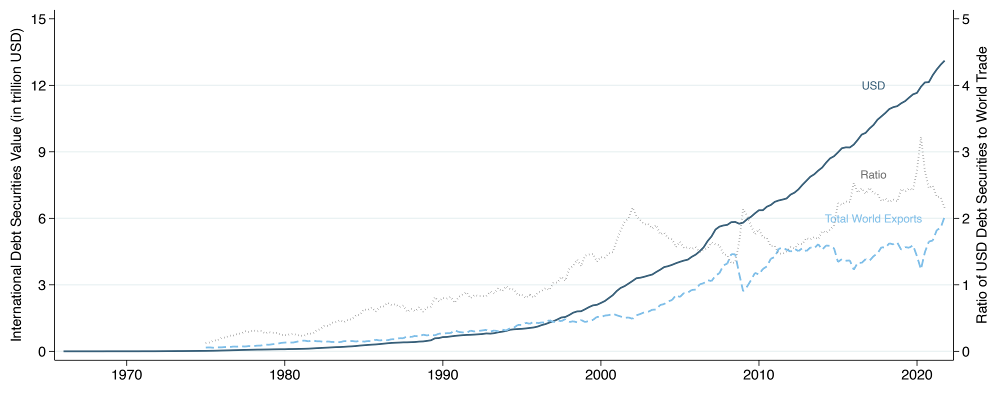
Our liquidity channel theory additionally explains why the volume of dominant currency debt is not capped by the volumes of world trade flows that are denominated in the dominant currency. Theories where complementarities arise from asset-liabilities mismatch (Gopinath and Stein 2021, Chahrour and Valchev 2022) generate a role for financial assets up to the amount of trade transactions, since the financial liabilities necessary to offset any balance sheet mismatch are bounded at those same volume. However, in today's dollar world, the volume of cross-border financial assets trade dwarfs the volume of real goods trade. Figure 4

[26] Early theoretical and empirical contributions on the vehicle currency role of the dollar in international trade include Krugman (1980), Rey (2001), and Goldberg and Tille (2008).

[27] One can also interpret our model to shed light on the dominance of exchange rate pegs to the dollar and foreign central bank accumulation of dollar reserves (Ilzetzki et al., 2019) via an analogous logic. By pegging the exchange rate, the central bank reduces currency mismatch costs to local borrowers, while enabling them to benefit from issuing in the liquid dollar market. In order to maintain the peg, the central bank accumulates dollar assets and trades these dollars to local firms for their goods' revenues when needed. The local firms then use the dollars to settle their dollar liabilities. The demand for these reserve assets coming from settlement needs generates a convenience yield on both public and private safe dollar claims.

[28] As of 1815, the British Empire was still relatively small with little presence in Africa, the middle east, and Asia outside of India. Its land mass and population as a share of the world did not reach their peaks until the early 20th century with most gains in the second half of the 19th. In addition, the largest expansion of trade in world GDP in the 19th century occurred in the 1880s, when it grew from approximately 5% to 10%.

Figure 4: **Values of cross-border financial claims in USD and total world exports**



Notes: This figure plots the total values of cross-border bank debt securities denominated in US dollars with the total values of world exports at a quarterly level, and the ratio of the two series.

plots the evolution of cross-border bank liabilities denominated in US dollars from 1975 until today relative to total world exports at a quarterly level. The average ratio in the last twenty years has been two. Accounting for the fraction of world trade that is denominated in US dollars (approximately 40% in recent years), the relative sizes are five-fold. Considering all debt securities in the world outstanding, the ratio of dollar-denominated financial claims to world trade annually has on average been eight-fold larger since 2015.

Relatedly, our theory implies particular normative prescriptions for governments whose aim is to promote their currency's internationalization: specifically, it suggests that opening up the capital account and deepening the size and liquidity of financial markets in the domestic currency is the primary necessary driver, rather than policies aimed at enhancing use of the domestic currency in global trade invoicing.

4.3 Indexed Versus Nominal Debt Denomination

The preceding examples highlight the unique role of the liquidity mechanism in driving denomination decisions across currencies. Our last example shows that the liquidity mechanism can also account for denomination choices *within* a given country. We focus on the decisions of private sector actors of whether to issue debt denominated in nominal or real units. From the perspective of our argument, this example has particular conceptual value since it allows us to study denomination decisions in a setting that fully abstracts from cross-border forces, including trade-related ones and differences in economic size.

Private debt is commonly denominated in nominal rather than real units even though consumption is real rather than nominal. We rationalize this empirical regularity by linking it to the fact that the safe government-backed asset in most economies is a nominal rather than a real bond. While the rationale for the sovereign to issue nominal debt has been discussed

in the literature (e.g., Bohn 1988; Doepke and Schneider 2006), there has been less attention to the choices of private sector issuers.

We develop this point by extending our model to include nominal considerations. Consider the within-country model, but let us now shift from real to nominal payoffs and allow for risks to the price level. Denote the price level as ρ_t so that the nominal value of the revenue of $\pi = 1$ at any date is ρ_t. We normalize the initial price level as one so that $\rho_0 = \rho_1 = 1$ and further assume that $E[\rho_2] = 1$. Firms choose between issuing inflation-indexed debt with face value of ρ_2 or nominal debt with face value of one.

We consider the case where there are L_N nominal government committed assets with liquidity λ_N. If a firm issues a nominal bond, the bond also has liquidity of λ_N. On the other hand we assume that inflation-indexed debt is completely illiquid with $\lambda_R = 0$.

The analysis of this extension closely follows that of the main model. A firm that issues nominal debt faces inflation risk in its date t_2 repayment. We parameterize the asset-liability mismatch risk as incurring a cost of K_i for firm i. The benefit to the firm is that it can use nominal bonds as liquidity if revenues are early at date t_1. Thus, the utility for nominal bond issuance and using bond liquidity at t_1 is,

$$U_N(m_{I,N}, m_{F,N}, K_i) \equiv \frac{\beta(1-\beta)}{2}\left[\lambda_N \left(m_{F,N} m_{I,N}\right)^{\theta-1}\left[m_{F,N} + \phi\, m_{I,N}\right] - K_i\right]. \tag{29}$$

which is the expression we developed in Equation (14), with some slight relabeling.

A firm that issues real debt sells the debt at price $\beta^2 E[\rho_2]$, which is β^2. If a firm is early and chooses to purchase a nominal bond with its early revenues, it gains the liquidity benefit from the bond but incurs mismatch risk since the debt repayment is promised in real terms. We again parameterize this cost as incurring K_i. Note that the benefit in this case is strictly lower than that of issuing nominal bonds, since in the latter case the firm also harvests the convenience yield when issuing the nominal liquid bond. Thus, the option of issuing real debt and storing liquidity in the nominal bond is dominated and we set it aside.

If instead the firm issues real debt and if early stores its goods, it obtains revenues of ρ_2 at date 2 which exactly repays the firms debt. The utility of this option is,

$$U_R(m_{I,N}, m_{F,N}, K_i) = 0. \tag{30}$$

This is because the investment cost is β^2 which is exactly covered by the proceeds from bond issuance. The firm also has exactly enough goods to repay its t_2 debt and avoid the mismatch cost of K_i.

Thus a firm chooses to issue nominal bonds if $U_N > U_R = 0$. Denote $\hat{K}$ as the cutoff cost for issuance of nominal bonds in (29). Then,

$$m_{I,N} = L_N + H(\hat{K})F \quad \text{and} \quad m_{F,N} = \phi H(\hat{K})F.$$

As in our baseline, this model features a complementarity in issuance decisions via liquidity: as $m_{F,N}$ and $m_{I,N}$ rise, U_N rises and the incentive to issue nominal bonds rises. A higher L_N or higher λ_N shifts the incentive towards nominal bond issuance.

TIPS and Treasury bills. Our theory rationalizes why US nominal Treasury debt is the safe asset and carries a convenience yield rather than CPI-indexed Treasury debt (TIPS) (Fleckenstein et al., 2014). If one considers safety in terms of owning an asset to purchase a given (real) consumption basket, it is puzzling that the safe asset in the US is not TIPS. However, if one shifts the terms, as our model does, to owning an asset to repay a nominal financial obligation, it becomes clear that nominal Treasury bills and reserves are an order of magnitude more liquid than TIPS and hence nominal Treasury (and other nominal safe assets) carry the convenience yield.

Moreover, the differences in relative liquidity also rationalize why the large majority of private debt contracts are denominated in dollars. The theory would predict that if the US government shifted the composition of its debt issuance to be primarily inflation-indexed debt (i.e., TIPS), then private debt denomination decisions would follow and also primarily be in real terms.

Greenback bonds. An empirical test of the theory arises in the historical context of the United States. While the predictions for TIPS versus Treasuries would require the US government to make the unlikely switch of its issuance of Treasury bills to TIPS, during the 19th century, the US government made a switch that is conceptually almost identical. For much of the 19th century, the US like other advanced economies of the day, operated on a metallic standard in which legal dollars had a claim to gold and/or silver. Government bonds were settled in gold, and the government also minted gold notes and coins. Thus, gold was the effective unit of denomination of all public and private debts.

However, during the Civil War and its aftermath, the US exited the gold standard and the government issued non-gold backed fiat money ("greenbacks"). During this time, the government also issued bonds that were not contractually paid in gold, and could be settled in greenbacks. As a result, we observe that the national banks of the era held these bonds and issued national bank notes, which traded at par with greenbacks. Corporate debt in the form of railroad bonds were also repayable in the legal tender of the time (i.e., greenbacks, see Friedman and Schwartz 1963).

The greenback experiment was a shift in the denomination of the liquid government asset. The fact that the private sector also shifted to denominating claims in greenbacks, rather than gold, provides support for the predictions of our theory. Eventually, in the late 19th century, the US returned to the gold standard, and government debt was again payable in gold. At this point, gold clauses that stipulated that contracts contained a claim to gold, which remained in place until the 1930s, were introduced into private contracts. The shift in private contract denomination into and out of gold can be explained via the liquidity force of the model.

5 Implications for International Monetary System

Our model also has several implications for rationalizing the architecture of the international monetary system. First, we show that the sovereign's ability and incentive to invest in institutions that deepen financial market liquidity are larger in the center country that hosts the dominant currency. These investments in turn generate additional complementarities that deepen dominance and can thereby explain the persistence of regimes. Second, we provide novel predictions for the impact of government debt issuance on firm issuance incentives. Third, we illustrate the complementary benefits to debt issuance in the cross-section of firms, which matches empirical patterns of issuance. Fourth, we consider welfare aspects of the model, in the context of the Bretton Woods era liquidity coordination. Lastly, we provide an extension that incorporates aggregate risk which rationalizes the existence of dominant currency central bank swap lines.

5.1 Endogenous Complementarities Between Financial Innovation and Currency Dominance

We have discussed how financial market liquidity leads to currency dominance. We now turn attention to a complementary mechanism through which dominant status endogenously gives the government of the dominant country further incentives to innovate in institutions and financial technologies that advantage its market liquidity. These innovations are less costly to finance in the dominant currency, and they further entrench dominance. The history of the Bank of Amsterdam's innovations in the florin, and the institutional changes in the London money market that impacted the pound sterling include several dimensions of these additional sources of complementarity.

We specify the sovereigns' objective functions. The government of country j maximizes:[29]

$$W_j = \underbrace{F_j \int u^F_{i,j}(K_i)\,\mathrm{d}H(K_i)}_{\text{Domestic firms' welfare}} + \underbrace{L_j(P_{0,j} - \beta^2)}_{\text{Seigniorage revenues}} . \tag{31}$$

The first term in this objective function corresponds to the purely utilitarian welfare criterion that aggregates the preferences of domestic firms. In addition to this standard utilitarian objective, we allow the government to have a profit motive, which is reflected in the second term of equation (31). These seigniorage revenues correspond to the convenience yield on sovereign debt issued at t_0, which scales linearly with the size of government issuance L_j. These revenues could be used to finance a costly investment in government-backed institutions such as ones that expand state or fiscal capacity.

For the rest of the analysis in the paper, we consider the case in which A is the higher

[29]Note that investors always break even in equilibrium, so that we omit the utility of investors in writing the sovereign objective.

liquidity market. We therefore focus on class BA equilibria—the only ones that remain present once asymmetry in fundamentals is sufficiently large. The analysis is of course symmetric for the case in which B is the higher-liquidity country. We now consider the sovereign's incentives to expand its commitment to government-backed securities (increasing L_j).

Proposition 3 (**Sovereign incentives**). *Consider an interior, stable equilibrium of the model of class BA, with a higher convenience yield in the dominant country A than in country B, and in which we hold seigniorage earned on existing (inframarginal) sovereign debt units fixed. There exists a foreign denomination threshold $\hat{K}^\dagger$ such that if foreign denomination in the dominant currency is sufficiently high ($\hat{K} > \hat{K}^\dagger$), the sovereign's incentives to invest in government commitment are larger in the dominant country:*

$$\frac{\partial W_A}{\partial L_A} > \frac{\partial W_B}{\partial L_B}. \tag{32}$$

The results for class AB equilibria are symmetric.

Proof. See Appendix Section A.3. □

In analyzing these incentives, we assume that the sovereign receives the seigniorage revenue stream from the convenience yield at issuance. In this sense, the seigniorage aspect of the sovereign's problem is akin to that of a durable goods monopolist (Coase 1972): the existing units of government debt are inframarginal and hence variation in the convenience yield on old issuance does not affect the revenues on them, but the choice of L_j does affect the seigniorage revenues earned on newly issued debt.[30]

The preceding proposition demonstrates that the government in the dominant country has a greater incentive to undertake costly investments in commitment technologies that allow it to expand L. For example, investing in the military strengthens the government's access to resources, thereby expanding fiscal capacity and the ability to issue safe assets. This occurs because of two forces. First, the sovereign in the dominant country faces a higher marginal value from additional debt issuance, in terms of seigniorage revenues, because of the higher convenience yield. Second, the non-dominant country has a lower incentive because its liquidity expansion does not benefit the set of inframarginal domestic firms that have already chosen to issue in the dominant currency. An additional source of complementarity, in practice, is that the government's ability to invest is also higher in the dominant country, because the convenience yields that the government earns generates a larger revenue stream that can reinforce the government commitment (Jiang et al., 2020).

[30]If we were to consider the case in which inframarginal debt is also re-priced (as for a sovereign with commitment and long planning horizons), there would be an additional tradeoff, as increasing L would expand the revenue base, while at the same time reducing the per-unit revenues earned on *all* units of debt, as in a Laffer curve. The strength of this mechanism would depend on the elasticity of demand for the sovereign bonds.

Historically during the period of pound dominance, the British government issued debt at the most advantageous terms of any sovereign, which gave it a funding advantage for military and naval growth. These investments enabled further imperial expansion that increased the government's tax base and access to resources, thereby expanding its fiscal capacity to issue more debt. Additional debt issuance then deepened the asymmetry of Britain's financial market depth relative to other countries', drawing in more firms into the pound equilibrium and lowering the costs of financing future investment for all domestic firms. This positive dynamic feedback loop leads to the endogenous persistence of dominant regimes.

As we discuss in the next section, financial development has taken the form of increases in government safe asset supply as well as improvements in the capacity of the private sector to issue safe assets. In Appendix Section A.5, we also consider an extension of the model in which we allow for innovations that increase the pledgeability of private revenues. As in the case with the government's commitment technology, we show that the incentive to invest in firms' revenue pledgeability is larger for the dominant country.

It is worth remarking on how this *endogenous incentives* mechanism relates to the hysteresis properties that typically emerge in dynamic multiple equilibrium models. In such settings, it is common for equilibrium persistence to arise: for instance, a typical approach (e.g., Cole and Kehoe 2000) is to assume that the sunspot variables resolving equilibrium multiplicity are persistent. Although we do not explicitly model transitional dynamics, our model also generates persistence: in a dynamic environment, the increasing returns to scale would readily generate historical path-dependence, making the model well-suited to explaining the strong persistence of dominant currency regimes. The endogenous incentives mechanism highlighted in this section is distinct from but complementary to these baseline hysteresis properties of the model, and by generating additional amplification, it further boosts equilibrium persistence.

5.1.1 Investments in Commitment Technologies Throughout History

Equipped with the theoretical result above, we now discuss several ways in which this interaction between currency dominance and incentives to invest and financially innovate have appeared throughout history.

Innovations and complementarities in Amsterdam. The Bank's introduction of a specie-florin repo facility in 1683 is credited with propelling the florin to global prominence (Quinn and Roberds, 2014a). Before this facility, obtaining florin required depositing specific coins, and accounts were primarily held by those with liquidity needs in florin-denominated bills of exchange.

The specie repo facility was an investment in financial technology that greatly expanded access to florins by allowing individuals and businesses to monetize safe but illiquid assets. The popularity of this facility, which doubled the quantity of florins at the Bank of Amsterdam

despite the fees charged to use it, also illustrates the complementarity in the issuance incentives that we model.[31] For specie investors, issuing florin against their assets was profitable because of the convenience yield, which captures the liquidity benefits of transacting in florin, reflecting the incentives to provide liquidity even by those without payment needs.

Bank of England's changing role. In the UK, the evolving role of the Bank of England from its creation in 1693 to the 19th century encapsulates the institution's changing incentives to facilitate financial market liquidity. At its founding, it was a private corporation that was granted several privileges in return for raising and administering the Crown's debt, and during the early part of its history, the Bank competed with other private banks such that it sometimes limited market liquidity in order to protect its own balance sheet.[32]

Beginning in the 1830s, a series of legislative reforms changed the Bank of England's role into one of a liquidity supplier. First, the Bank's notes became legal tender for pound sterling debts in 1833, which legally expanded the supply of pounds sterling.[33] Second, the entire note issuance was consolidated onto the Bank's balance sheet and given a large fiduciary issue in 1844, which again expanded the supply of pound-denominated settlement instruments. Third, by the mid-19th century, the Bank of England established its role as a reliable lender of last resort to the financial sector (an innovation that Bagehot (1873) credits to the Bank), where its balance sheet explicitly became the backstop to the private bills market.

Innovations and complementarities in the London money market. Institutionally, the private bills market also underwent changes. The legal codification of the contractual terms for bills of exchange coordinated the market on the terms of borrowing and the procedures for default, which reduced their information sensitivity (Dang et al., 2017) and collectively raised the safety and liquidity of the London money market. The growing clarity on the Bank of England's discount window rules also helped to homogeneize money market securities and raised the incentives to produce high quality "discountable" bills.

The government lowered the costs for the banking sector to create bills of exchange by deregulating private banks' equity issuance in 1830. This fueled British banks' overseas expansion, which took advantage of the growth in world trade during this period. London

[31] The quantity of florin doubled from approximately eight million to sixteen million from the mid 17th to the beginning of the 18th century and is credited with drawing Europe's specie trade to Amsterdam, where it could be more profitably be conducted.

[32] The privileges restricted banking competition and gave the Bank of England a monopoly over note issuance. From 1697 until 1844, only the Bank could raise equity; all other banks were restricted to partnerships of six or fewer (after 1844, this was altered to a radius of 25 miles around London). In 1708, the Bank was granted an exemption to laws restricting bank note issuances to private partnerships (Broz and Grossman, 2004). The Bank's abuse of its monopoly during the 1825 crisis led to the Banking Act of 1826, which mandated Treasury monitoring of small-denomination note issuance.

[33] Like most currencies during this era, the pound sterling referred to a specific metallic coin, and obligations denominated in sterling were contracted to be repaid in those coins. However, coins were inconvenient for the reasons already discussed, and private banks like the Bank of England found it profitable to issue paper notes denominated in sterling (i.e., claims on sterling coin).

banks pursued a business model of issuing bills of exchange collateralized on the large base of international trade, which simultaneously allowed them to capture the dual benefits of issuance: the convenience yield for creating money-like assets in London and the settlement benefits from providing credit to exporters around the world (Xu, 2022). Their success relative to competitors from other nations was due to their access to the London bills market where they re-discounted the bills they underwrote. As in Amsterdam, these private and government investments in commitment and pledgeability monetized a pool of previously illiquid assets, further increasing market depth in the London money market.

Innovations and complementarities in the dollar market. Innovation in the US financial system has allowed privately-issued short-term debt instruments to add to the pool of dollar-denominated money market liquidity. The growth of the US banking system and of the commercial paper market are early examples of these liquidity-producing financial technologies. In more recent decades, securitization (Mian and Sufi 2009, Keys et al. 2010) allowed for further private-sector production of safe liquid assets, while the expansion of repo markets (Gorton and Metrick 2012, Krishnamurthy et al. 2014) broadened liquidity in the overnight segment of dollar money markets. Our study demonstrates that the financial and central banking advancements in the United States are a natural consequence of the incentives that come with being the issuer of a dominant currency, and that these developments further reinforce the currency's dominance.

5.2 Asset Pricing Channels and Dominance

In addition to matching the empirical evidence on quantities of debt issued in the dominant currency, our model also speaks to the relationship between asset prices, issuance, and dominance. The following proposition characterizes these asset pricing results.

Proposition 4 (**Asset pricing**). *The convenience yields in the model satisfy the following properties in class BA equilibria (the results for class AB equilibria are symmetric):*

1. *Holding fixed country fundamentals, foreign denomination impacts convenience yields positively:*

$$\frac{\partial(P_{0,A} - \beta^2)}{\partial \hat{K}} > 0. \tag{33}$$

   A corollary is that, comparing across equilibria for a given set of parameters, equilibria with higher foreign denomination in currency A feature higher convenience yields in country A.

2. *Increasing government debt supply L_A impacts the convenience yield $P_{0,A} - \beta^2$ in two opposite ways: the resulting increase in liquidity demand ($m_{F,A}$) affects the convenience*

yields positively, while the resulting increase in liquidity supply ($m_{I,A}$) affects it negatively. Hence, depending on the relative strength of these two channels, the model can generate either a net negative impact of L_A on convenience yields or a net positive one.

Proof. See Appendix Section A.3. □

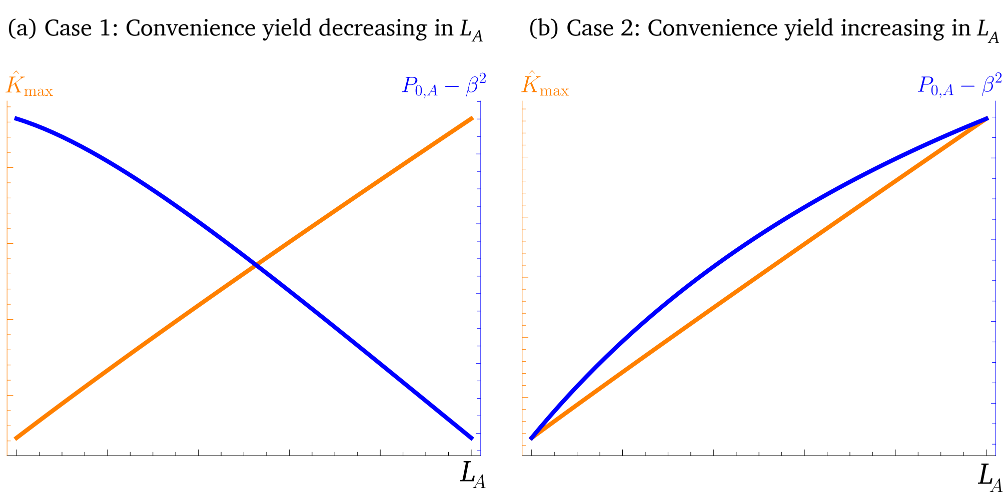
The first part of Proposition 4 stresses that conditional on holding country fundamentals fixed, stronger dominance of the center country, as captured by foreign denomination in its currency, raises its convenience yield. In addition, comparing across equilibria, those equilibria which feature more foreign denomination will have higher convenience yields.

On the other hand, as shown in the second part of the proposition, increasing the stock of sovereign debt L, which alters fundamentals, can have ambiguous overall impact due to the presence of two opposing forces. Recall the expression for the convenience yield in country A, which is

$$P_{0,A} - \beta^2 = \frac{\lambda_A \beta(1-\beta)}{2} m_{F,A}^{\theta} m_{I,A}^{\theta-1}. \tag{34}$$

When $\theta < 1$, both liquidity demand ($m_{F,A}$) and liquidity supply ($m_{I,A}$) impact the convenience yield: intuitively, rising liquidity demand increases the convenience yield, while rising liquidity supply reduces it. Figure 4a illustrates this point for a parametric case where $\theta = 0.9$: while foreign denomination is increasing in sovereign debt supply L_A, the convenience yield is decreasing in L_A. This relationship matches the empirical findings in Krishnamurthy and Vissing-Jorgensen (2012) that convenience yields are *decreasing* in government debt supplies.

Figure 5: **Convenience yields and government bond supply**



Notes: We focus on equilibria of the class BA, in which B firms switch to currency A. We show the behavior of the convenience yield $P_{0,A} - \beta^2$ for currency A as a function of government debt supply L_A. The analyses show these simulated comparative statics selecting the stable equilibrium with maximum foreign denomination threshold, $\hat{K}_{\max}$, which is also plotted. Panel A uses $\theta = .9$, while Panel B uses $\theta = 1$. The rest of the parameter values in this example are $F_A, L_A, L_B, \lambda_A, \lambda_B = 1$; $\phi, \underline{K} = .5$; $\alpha = 1.5$; and $F_B = .1$, where we set F_B low to emphasize the impact of liquidity supply on convenience yields, minimizing the indirect foreign denomination effect.

The $\theta = 1$ case is a knife-edge parameter configuration in which the liquidity supply channel is shut off: in this special case, shifts in the liquidity supply schedule (such as changes in government debt supply L_A) do not have a direct effect on bond prices, but rather only impact them through their indirect effect on stronger equilibrium foreign denomination $\hat{K}$. Therefore, the convenience yield is unambiguously increasing in government debt supply L_A. Figure 4b illustrates this case where both the convenience yield (in blue) and foreign denomination, $\hat{K}_{\max}$ (in orange), are increasing functions of L_A.

It is worth noting that the asset pricing channels of our model indicate that it is possible, if L_A is sufficiently large relative to L_B, to have stable equilibria where A has lower convenience yields, but it is nonetheless still dominant as a currency of denomination. In such a case, the issuance in currency A is sustained not by the convenience yield force in our model, but rather by the settlement benefit force, which is much higher in the dominant country. These two forces therefore allow our theory to accommodate the empirical finding that, according to certain measures in certain periods of time, US Treasuries may attain lower convenience yields than government bonds in other currencies (Diamond and Van Tassel, 2021).

5.3 Issuance and Complementarity in the Cross Section of Firms

In the baseline set of international equilibria outlined so far, firms trade off the costs of currency mismatch with both of the two benefits of dominant-currency issuance: favorable convenience yields and high settlement liquidity. In reality, issuers in the cross-section of firms may benefit disproportionately from one or the other incentive.

In this section, we present an extension of our baseline model that separates out these two roles according to the separate issuance incentives and formalizes the complementary nature of the denomination decisions of these two types of corporate borrowers. Hence we provide a positive explanation for why, today and as well as historically, we observe a wide range of different borrowers issuing debt in the dominant currency—from the safest global borrowers all the way down to more speculative and risky ones. We also illustrate how these forces respond in heterogeneous ways to an increase in government debt issuance such that the strategic complementarities between these actors reinforce the dominant currency equilibrium.

We observe, for example, that global corporate borrowers with poor credit ratings issue debt in the dominant currency even though their debt is not sufficiently money-like to benefit from convenience yields. Nonetheless, these borrowers will still be attracted to issuing in the dominant currency because of its settlement liquidity. As such, these issuers act as net *liquidity demanders* in the dominant-currency money markets. On the other hand, particularly safe borrowers such as the German sovereign-backed issuer KfW may not have many settlement needs but still be drawn to dominant currency issuance primarily to capture the convenience yield, thereby acting as net *liquidity suppliers*.

Our analysis focuses on the *class BA* equilibria and separates the liquidity demand and

liquidity provision roles in the cross-section of firms in country B. Specifically, we now allow the overall mass of firms F_B to be composed of two different groups of firms:

- A first mass F_B^+ consists of *pure liquidity suppliers*: these are issuers for whom $\phi = 0$, which therefore never experience an early realization profits and hence have no motive for demanding liquidity and will not contribute to the liquidity-demander masses $m_{F,j}$ in either country. The issuance incentive is purely a function of the convenience yield in each country:
$$u_{i,j}^{F+} = \frac{\lambda_j \beta(1-\beta)}{2} m_{F,j}^{\theta} m_{I,j}^{\theta-1} \tag{35}$$
- A second mass F_B^- consists of *pure liquidity demanders*. These are firms whose bonds have no possibility of re-sale in the money market of date t_1, so that effectively λ_j (which is now heterogenous for different assets) is zero for these firms' issues. These firms therefore will not contribute to the liquidity-supplier masses $m_{I,j}$. The issuance incentive is purely a function of the settlement benefit in each country:
$$u_{i,j}^{F-} = \frac{\lambda_j^+ \beta(1-\beta)}{2} \phi m_{F,j}^{\theta-1} m_{I,j}^{\theta} \tag{36}$$

The cost K_i follows the same distribution $H(K)$ in these two subgroups of firms, and $F_B = F_B^+ + F_B^-$. These two groups of firms will now have two different endogenous equilibrium thresholds $(\hat{K}^+, \hat{K}^-)$: liquidity suppliers issue in foreign currency if and only if $K_i < \hat{K}^+$, while liquidity demanders issue in foreign currency if and only if $K_i < \hat{K}^-$.

The equilibrium conditions pinning down the two thresholds of $(\hat{K}^+, \hat{K}^-)$ are:

$$\lambda_A m_{F,A}^{\theta} m_{I,A}^{\theta-1} - \hat{K}^+ = \lambda_B m_{F,B}^{\theta} m_{I,B}^{\theta-1}, \tag{37}$$
$$\lambda_A \phi m_{F,A}^{\theta-1} m_{I,A}^{\theta} - \hat{K}^- = \lambda_B \phi m_{F,B}^{\theta-1} m_{I,B}^{\theta}. \tag{38}$$

To complete the characterization of the equilibrium, the liquidity-demand masses in the two countries are

$$m_{F,A} = \phi \left[F_A + H(\hat{K}^-) F_B^- \right], \qquad m_{F,B} = \phi \left[1 - H(\hat{K}^-) \right] F_B^-, \tag{39}$$

while the liquidity-supply masses are

$$m_{I,A} = F_A + L_A + H(\hat{K}^+) F_B^+, \qquad m_{I,B} = G_B + \left[1 - H(\hat{K}^+) \right] F_B^+. \tag{40}$$

First, we illustrate the complementary nature of issuance by each type of firm. Consider the extreme case in which there is no mass of private liquidity suppliers (F^+). If there is also no government debt $L_j = 0$, the mass of liquidity supply $m_{i,j}$ will be zero, and there will be no settlement benefits for liquidity demanders (equation 36). Since these firms do not benefit

from the convenience yield, there is no incentive to issue debt, and $F^+ = 0$.

On the other extreme, consider the case where there is no mass of liquidity demanders (F^-), meaning that $m_{F,j} = 0$. Without demand for settlement, the mass of circulating medium $m_{I,j}$ serves no purpose and therefore does not harvest a convenience yield. As a result, there is no utility to issuance by either type of firm and no private debt issuance. The issuance decisions of the two types of firms are therefore complementary in the cross-section, as formalized in the following proposition.

Proposition 5 (Cross-sectional complementarities). *The equilibrium foreign denomination decisions of liquidity suppliers (F_B^+) and liquidity demanders (F_B^-) are complementary:*

$$\frac{\partial \hat{K}^+(\hat{K}^-)}{\partial \hat{K}^-} \geq 0, \quad \frac{\partial \hat{K}^-(\hat{K}^+)}{\partial \hat{K}^-} \geq 0, \tag{41}$$

with the inequalities strict in an interior equilibrium (i.e., one which features positive foreign denomination by both types of firms). Hence strategic complementarity in the denomination decisions of these two classes of borrowers reinforces the dominant-currency equilibrium.

Proof. See Appendix Section A.3. □

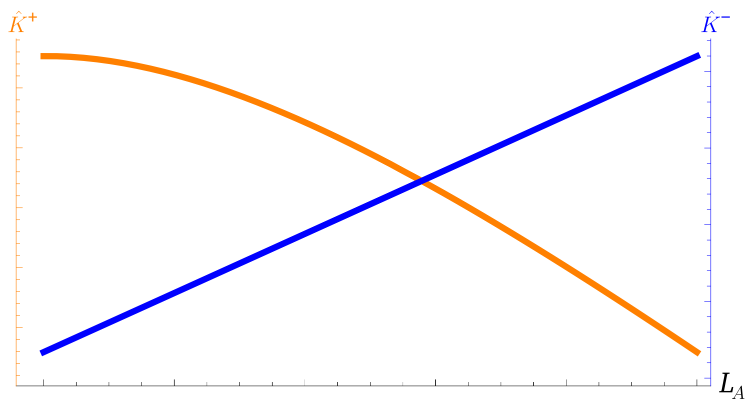
Second, we analyze the heterogeneous impact of increasing government debt on the issuance incentives of these two types of firms. A positive supply of government debt ($L_j > 0$) contributes to the liquidity supply $m_{I,j}$ that circulates as a medium of exchange and is useful for settlement. The settlement benefit then raises the utility of debt issuance for the liquidity demanders, leading to the creation of liquidity demand (F^-) in the same denomination. However, in the general case of $\theta < 1$ where government issuance reduces convenience yields, private liquidity suppliers that only benefit from the convenience yield (F^+) are *crowded out* (e.g., Roberds and Velde, 2016; Krishnamurthy and Vissing-Jorgensen, 2015). This crowding out force appears in a broad class of models in which the private sector incentive to create debt comes only from the convenience yield (e.g., Krishnamurthy and Vissing-Jorgensen, 2015; Gopinath and Stein, 2021; Gorton and Ordonez, 2022) and therefore only benefits firms that can create safe assets. In these papers, increasing the supply of dollar-denominated US government debt reduces private dollar debt issuance, thereby hampering the dominant equilibrium.

Our model also features an additional force where government issuance simultaneously raises the benefit of settlement such that private liquidity demanders are *crowded in*. As a result, the dominant equilibrium is sustained by the overall foreign denomination by both type of firms (Proposition 4), even though the foreign denomination by liquidity suppliers is partially offset by the impact on asset prices.

Figure 6 provides a parametric example illustrating the crowding-in and crowding-out effects. We plot the foreign denomination thresholds of liquidity demanders ($\hat{K}^-$) and liquidity

suppliers ($\hat{K}^+$) as we vary L_A with $\theta = .9$, keeping the other parameters the same as in Figure 4a. Foreign denomination by risky firms (liquidity demanders) increases with L_A, since these firms benefit by the improved liquidity of A markets but do not care about convenience yields. On the other hand, foreign denomination by safe firms (liquidity suppliers) falls with L_A, since these firms do not value the settlement liquidity benefits, but rather only care about the declining convenience yield.

Figure 6: **Crowding in and crowding out in the model with firm heterogeneity**



Notes: We consider comparative statics for the model with additional firm heterogeneity. We plot the foreign denomination thresholds for liquidity suppliers ($\hat{K}^+$) and liquidity demanders ($\hat{K}^-$) as a function of government debt supply L_A. The analyses show these simulated comparative statics selecting the equilibrium with highest foreign denomination. We use the same parameters in Figure 4a, with $\theta = .9$. We set $F_B^+ = F_B^- = .05$, so that the total mass of B firms is equal to the one used in Figure 5.

5.4 Liquidity Coordination and Welfare

At various points in history, there has been the recognition that one country may not be able to sustain the creation of enough liquid assets to support the pace of real economic growth. Triffin (1978) provided a prominent discussion of this concern in the context of gold-backed dollar reserves in the Bretton Woods period. Our model allows us to discuss liquidity provision policy and welfare from the perspective of a global planner. We continue to consider the case in which A is the higher liquidity market, focusing on class BA equilibria and on the case where $\theta = 1$. We consider a planner whose objective is utilitarian over the preferences of the two governments:

$$\mathcal{W} = W_A + W_B. \tag{42}$$

Our first result is that the planner's choice of optimal foreign denomination features more foreign denomination in currency A than the competitive equilibrium.

Proposition 6 (**Global welfare**)**.** *Consider the case $\theta = 1$. Let K^* be the value of $\bar{K} \in [\underline{K}, \infty)$ that maximizes global welfare $\mathcal{W}$, and let $\hat{K}_{\max}$ be the stable interior equilibrium point*

featuring highest foreign denomination in the dominant currency.[34] *It holds that*

$$K^* > \hat{K}_{\max}. \tag{43}$$

Proof. See Appendix Section A.3. $\square$

At the heart of this result is the liquidity externality discussed in Section 2: foreign denomination in currency A by firms in country B carries social benefits in terms of improved market thickness, which are in excess of the private benefits of foreign denomination. Since these excess liquidity benefits are not internalized by the firms, there is too little private foreign denomination in equilibrium.

Conceptually, our setting presents an analogy with the theory of *natural monopoly* (Posner 1978). Here the higher-liquidity country (A) has aspects of a natural monopolist, since consolidating issuance in its currency is welfare-improving from a global perspective. An important difference between our model and theories of natural monopoly, however, is that in this setting first-best equilibria are also interior, as in Eichengreen et al. (2017). The planner's chosen equilibrium K^* achieves the *first best*, which can always be made into a Pareto improvement relative to the private equilibrium $\hat{K}_{\max}$ by introducing appropriate transfers.[35] Similarly to the normative results that feature in theories of natural monopoly, optimal policy in this setting will therefore feature a subsidy to foreign denomination in currency A.

In the results above we have considered the foreign denomination problem from the perspective of the global planner. A second, related question is how the shadow value of increasing government debt supply L_A in the leader country differs when viewed from the perspective of the global planner versus the sovereign in country A. That is, we are interested in comparing the two quantities $\frac{\partial \mathcal{W}}{\partial L_A}$ and $\frac{\partial W_A}{\partial L_A}$. If the shadow value from the global planner's perspective, $\frac{\partial \mathcal{W}}{\partial L_A}$, is higher, then the planner will prefer to increase L_A even beyond what is privately optimal for country A's government, leaving open the possibility of welfare-improving international coordination in sovereign liquidity provision.

Proposition 7 (**Liquidity coordination**). *Consider the case $\theta = 1$. In a stable, interior, class BA equilibrium, the shadow value of increasing L_A is higher from the global planner's perspective, as compared to the perspective of the sovereign in the leading country (A), if and only if the following is satisfied:*

$$H(\hat{K})\frac{\lambda_A}{\lambda_B} > \frac{1}{2}\frac{L_B}{F_B} + \left[1 - H(\hat{K})\right] \quad \iff \quad \frac{\partial \mathcal{W}}{\partial L_A} - \frac{\partial W_A}{\partial L_A} = \frac{\partial W_B}{\partial L_A} > 0. \tag{44}$$

The results for class AB equilibria are symmetric.

Proof. Differentiate W_A, W_B with respect to L_A, and rewrite to yield the inequality. $\square$

[34] In the illustration in Figure 2, this corresponds to equilibrium point 2 when considering class BA equilibria.

[35] We note that utility is transferable in our setting owing to the quasi-linear structure of preferences.

In the expression above, note that if A is dominant, then $H(\hat{K})$ is high, tending to one, while $1 - H(\hat{K})$ tends to zero, and likewise the ratio $\frac{\lambda_A}{\lambda_B}$ is high, increasing the likelihood that this condition is satisfied. The direction of this result therefore hinges on the relative magnitudes of L_B and F_B. Improving liquidity in country A has two effects on welfare in country B: on the one hand, it improves the utility of the infra-marginal B firms that have already switched to foreign currency, but on the other hand it reduces the convenience yields earned on sovereign issuance L_B by inducing higher foreign denomination. If country B has large private borrowing needs relative to the stock of safe government debt outstanding, the first effect outweighs the second, so that increasing L_A is also welfare-improving from the perspective of country B.

If the condition in equation (44) is satisfied, the global planner will want to engineer incentives for country A to further increase its liquidity supply, financing these with transfers from country B. These results provide a lens to interpret historical international liquidity provision arrangements, such as the Bretton Woods agreements of 1944. In the Bretton Woods system, major world economies effectively coordinated on having liquidity provided by the United States with the bulk of the gold reserves underpinning the Bretton Woods gold standard held at the Federal Reserve.

Throughout the Bretton Woods period, the United States held more than 90 percent of the world's gold reserves (Monnet and Puy 2020), with large transfers of gold from central banks outside of the US to New York during and after WWII. These transfers and the resulting coordination on a US-backed gold convertibility system provide a historical counterpart to the possibility of welfare-improving coordination in international liquidity provision that features in our model. In response to the classic Triffin (1978) dilemma that the US gold reserves would be insufficient to back its internationally held liabilities, our model would have prescribed more liquidity coordination in the form of transfers of commitment (i.e., gold) to the US.

5.5 Aggregate Risk, State-Contingent Liquidity, and Default Risk

Our discussion of liquidity provision did not draw a distinction between state-contingent and non-contingent expansions of liquidity supply L_A, although state contingency also played an important role within the Bretton Woods architecture—for instance, through the role of central bank swap line arrangements, which remain a core feature of the international monetary system to this day. A formal discussion of this topic requires extending the model to incorporate a role for aggregate risk, and hence we now suppose that ϕ is subject to an aggregate shock, realized at time t_1. The state is $\omega \in \Omega$, and in state ω, which has probability q_ω, the early profit realization probability ϕ takes on the value ϕ_ω. The state realization is a shock to aggregate liquidity demand: if the realized value of ϕ is higher, more firms experience timing mismatch and therefore there is more overall demand for liquidity.

We analyze this extended version of the model in the $\theta = 1$ case. We allow for state-

contingency in the supply of government assets L^A in the leading country, but we assume that L_B, F_A, and F_B are all not state-contingent. The value of L_A in each state ω is L^A_ω. The following result presents the equilibrium indifference condition in this extended model.

Proposition 8 (**Aggregate risk**). *Consider the case $\theta = 1$. In the model with aggregate risk, focusing on the case in which A is the dominant currency (class BA equilibria), the equilibrium condition that determines the marginal firm i with $K_i = \hat{K}$ is*

$$\lambda_A\Bigg(\mathbb{E}[\phi_\omega](2(F_A + H(\hat{K})F_B) + \mathbb{E}[L^A_\omega]) + \mathrm{Cov}[\phi_\omega, L^A_\omega]\Bigg) - \hat{K} = \lambda_B \mathbb{E}[\phi_\omega]\Bigg(2(1 - H(\hat{K}))F_B + L_B\Bigg). \tag{45}$$

The results for class AB equilibria are symmetric.

Proof. See Appendix Section A.4. □

With the addition of aggregate risk, the equilibrium condition is nearly the same as in the baseline case, except for the additional terms $\mathbb{E}[L^A_\omega]$ and $\mathrm{Cov}[\phi_\omega, L^A_\omega]$, which are respectively the expected value of L^A and its covariance with the timing mismatch probability ϕ_ω across states. The intuition for the expectations term is straightforward: higher average government debt supply L^A_ω shifts the left-hand side of this equality upwards and thereby increases the equilibrium foreign denomination threshold $\hat{K}$.

The covariance term is positive when liquidity supply L^A_ω increases in states of the world with high liquidity demand (and hence high ϕ_ω). When the covariance is zero, even with stochastic ϕ_ω, the model collapses to the baseline case with constant ϕ. When L^A_ω is positively correlated with ϕ_ω, the extra covariance benefit makes currency A more attractive.

Central bank institutions such as the discount window and swap lines arrangements achieve positive covariance: both technologies allow the dominant currency issuer to expand liquidity supply when liquidity demand is particularly high.[36] Swap lines provide emergency liquidity supply by one central bank to others: overwhelmingly, the central bank on the supplying end of these arrangements has been the US Federal Reserve, which provided US dollar liquidity in overseas markets during stress periods such as the global financial crisis of 2008-09 and the COVID crisis of 2020. These state-contingent expansions of dollar-denominated money market liquidity increases the expected liquidity benefits from issuing dollar debt and reinforces the dollar dominance equilibrium. As a result, they can be an important part of the architecture of the international monetary system (Bahaj and Reis, 2021).

The analysis also allows us to discuss default risk in government debt and the importance of the safety of the settlement asset. From the standpoint of t_0, default risk is that L^A_ω can fall in some states of the world. One effect of this risk is to reduce $\mathbb{E}[L^A_\omega]$, thereby reducing government supply and foreign denomination $\hat{K}$. A second effect is that if risk is higher

[36]It is worth noting that the innovation of a central bank acting as a lender of last resort occurred in London during the period of British pound dominance while the latter has been an innovation of the Federal Reserve system during dollar dominance.

in high-liquidity states of the world, then the covariance term is negative, further reducing effective government supply and $\hat{K}$. Hence higher perceptions of sovereign default risk hinder a currency's internationalization.

These effects concern risk realized between t_0 and t_1. There is a third effect that concerns risk realized between t_1 and t_2 that does not arise in the modeling of this section, but is likely an important concern. Bonds purchased at t_1 as settlement instrument for private debt at t_2 are poor settlement instruments if these bonds carry default risk. For example, if between t_1 and t_2, the bond defaults completely, then the firm will not own an asset to extinguish its debt. At t_0, default risk in government debt will mean that the currency is a less attractive currency in which to denominate private debts.

6 Conclusion

Our theory of the liquidity force that generates and sustains currency dominance rationalizes the historical emergence, persistence, and transitions between regimes over the last four centuries. The key feature that distinguishes the Dutch florin, the British pound, and the US dollar during their respective periods of dominance is the depth of the financial markets in those currencies. The historical evidence emphasizes that economic size and trade volumes themselves are not necessary to either generate or to sustain dominance. Instead, the key innovations have been financial. Whether financial depth was intentionally created through investing in technologies, as in the Dutch case, or it was the outcome of global disruptions in geopolitical power as in the British and American cases, a large homogeneous pool of safe government-backed securities has been the springboard for the money market in each dominant currency. Once dominance is established, the ability and incentives to invest in institutions that facilitate financial market depth are greater in the center country and lead to greater entrenchment of the dominant regime.

An often-asked question in academic and policy discussions is whether the Chinese renminbi might be poised to displace the US dollar as the world's dominant international currency in the near future (Horn et al. 2021, Clayton et al. 2022). Our theory suggests that China's safe debt markets are currently not sufficiently liquid, safe, or investable in size to challenge the dollar's status.

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Online Appendix for
"Liquidity, Debt Denomination, and Currency Dominance"

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July 2024

A Additional Microfoundations and Proofs

A.1 Alternative Microfoundation for Switching Cost K_i

In the model, the cost of converting your A good to A bonds is liquidity and the chance that you dont meet a trading partner. The trade that is needed for a B firm is to exchange B goods for A bonds. Suppose that involves both a fixed cost K_i and the liquidity probability. We can for simplicity assume that the exchange rate is one so its a purely financial cost incurred when contacting the traders in the A bond market. This modeling seems isomorphic to our current model. It also works to rule out the trade of a B firm converting to A bonds to store liquidity and then at maturity converting back to B to repay bonds. The cost benefit is relative liquidity vs K_i, which is the same as the trade we analyze. The story would that for a large B firm this contact is easy and so K_i is low, while for a small B firm its harder. Note that the contact could be, and perhaps is most natural, to think of contacting a trader in the onshore A country; but it could also be a trade with an offshore owner of B bonds.

A.2 Conditions for Equilibrium Existence and Convexity

This section provides a proof for the equilibrium characterization given in Proposition 1. We discuss the conditions required for equilibrium existence and stability, as well as the second-order conditions associated with the optimization problems that feature in the model. We show that under the Pareto form for the distribution $H(K_i)$ featured in our baseline parametric analyses, the model's objective function is well-behaved.

Consider equilibria of class BA (as the analysis is symmetric for AB equilibria). The marginal firm with $K_i = \hat{K}$ satisfies

$$\lambda_A \left(m_{F,A} m_{I,A}\right)^{\theta-1} \left[m_{F,A} + \phi\, m_{I,A}\right] - \hat{K} = \lambda_B \left(m_{F,B} m_{I,B}\right)^{\theta-1} \left[m_{F,B} + \phi\, m_{I,B}\right], \tag{A.1}$$

while the market clearing conditions are

$$m_{F,A} = \phi \left[F_A + H(\hat{K}) F_B\right], \qquad m_{F,B} = \phi \left[1 - H(\hat{K})\right] F_B, \tag{A.2}$$

and

$$m_{I,A} = L_A + F_A + H(\hat{K}) F_B, \qquad m_{I,B} = L_B + \left[1 - H(\hat{K})\right] F_B. \tag{A.3}$$

Define

$$\Delta(\hat{K}) = \lambda_A \left(m_{F,A} m_{I,A}\right)^{\theta-1} \left[m_{F,A} + \phi\, m_{I,A}\right] - \lambda_B \left(m_{F,B} m_{I,B}\right)^{\theta-1} \left[m_{F,B} + \phi\, m_{I,B}\right] - \hat{K}. \tag{A.4}$$

The equilibrium condition is then that

$$\Delta(\hat{K}) = 0. \tag{A.5}$$

Since $\Delta > 0$ implies a positive foreign denomination incentive, a stable equilibrium occurs when $\Delta(\hat{K})$ has a root which crosses zero from above (so that $\Delta' < 0$ at the root). Note that the function $\Delta(\hat{K})$ becomes negative for large $\hat{K}$:

$$\lim_{\hat{K}\to\infty} \Delta(\hat{K}) < 0. \tag{A.6}$$

Now consider two cases:

1. If the parameters Θ are such that $\Delta(\underline{K}) > 0$, then the existence of at least one interior stable equilibrium is guaranteed by the intermediate value theorem, given that $\Delta(\hat{K})$ is continuous.

2. If the parameters Θ are such that $\Delta(\underline{K}) \leq 0$, then a corner equilibrium occurs at $\hat{K} = \underline{K}$, which is stable if the inequality holds strictly (if $\Delta(\underline{K}) < 0$) and unstable otherwise (if $\Delta(\underline{K}) = 0$). Additionally, at least one interior stable equilibrium exists if and only if $\Delta(\hat{K})$ attains a positive value for at least some $\hat{K}$, again by the intermediate value theorem.

In the second case, in which $\Delta(\underline{K}) \leq 0$, the existence of an interior stable equilibrium then hinges on whether $\Delta(\hat{K})$ attains a positive value at some point over its domain $(\underline{K}, \infty)$. A necessary condition is that $H(\cdot)$ is sufficiently curved for low values of $\hat{K}$.

To see that the Pareto distribution, which we use in our baseline parametric analyses, can satisfy the necessary curvature condition (in a parameter-dependent way), it suffices to give an example. Figure A.1 shows such an example where the Pareto form attains a positive value for $\Delta(\hat{K})$ given $\Delta(\underline{K}) < 0$. The example uses symmetric country fundamentals: it is then clear that an interior, stable class AB equilibrium exists as well, in addition to the stable class BA equilibrium. This demonstrates that stable equilibria of all three classes outlined in Proposition 1 (interior class BA, interior class AB, and no foreign denomination) can exist simultaneously for a given parameter vector Θ, as desired.[1]

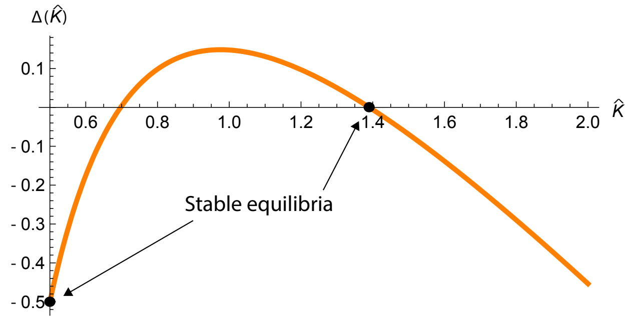
To gain an understanding on the conditions on $H(\cdot)$, it is also helpful to study the $\theta = 1$ case, in which it is possible to provide an analytical characterization. When $\theta = 1$, we obtain that:

$$\Delta(\hat{K}) = \underbrace{\phi\lambda_A \left[2F_A + L_A\right] - \phi\lambda_B \left[2F_B + L_B\right]}_{(1)} + \underbrace{2\phi F_B(\lambda_A + \lambda_B) H(\hat{K})}_{(2)} - \hat{K}. \tag{A.7}$$

Under strictly stronger A fundamentals ($F_A > F_B, L_A > L_B, \lambda_A > \lambda_B$), the term marked (1) is positive and constant. The term marked (2) is zero at $\hat{K} = \underline{K}$, and it asymptotes to $2\phi F_B(\lambda_A + \lambda_B) > 0$ as $\hat{K}$ goes to infinity. The second term is also strictly increasing.

[1] The properties in this example can also be obtained with asymmetric country fundamentals: for instance, small perturbations of the parameters in Figure A.1 will continue to yield double crossings of the horizontal axis.

Figure A.1: **Existence and stability: numerical example**



Notes: This figure plots the function $\Delta(\hat{K})$ taking $H(\cdot)$ as a Pareto distribution, and for the following choice of parameters with symmetric country fundamentals: $F_A = F_B = L_A = L_B = \lambda_A = \lambda_B = 1$, $\theta = .95$, $\underline{K} = .5$, $\phi = .5$, $\alpha = 1.5$. The black dots correspond to stable equilibria (no foreign denomination and interior of class BA). By symmetry, an interior class AB equilibrium also exists.

A necessary condition on $H(\cdot)$ for $\Delta(\hat{K})$ to attain a positive value (assuming the case $\Delta(\underline{K}) < 0$) is then that

$$\frac{d}{d\hat{K}}\left(2\phi F_B(\lambda_A + \lambda_B)H(\hat{K})\right) > 1, \tag{A.8}$$

or

$$2\phi F_B(\lambda_A + \lambda_B)h(\hat{K}) > 1 \tag{A.9}$$

for low values of $\hat{K}$. Candidate distributions are ones for which

$$h(\underline{K}) > \frac{1}{2\phi F_B(\lambda_A + \lambda_B)}, \tag{A.10}$$

and where $h(\cdot)$ is uniformly decreasing. The Pareto distribution satisfies these criteria, as do many other distributions.

A.3 Proofs Not Included in Main Text

This section contains proofs that are omitted from the main text.

Proof of Proposition 2 (Comparative statics). Define

$$\tilde{\Delta}(\hat{K}) = \underbrace{\lambda_A \left(m_{F,A} m_{I,A}\right)^{\theta-1}\left[m_{F,A} + \phi\, m_{I,A}\right]}_{\equiv u_A} - \underbrace{\lambda_B \left(m_{F,B} m_{I,B}\right)^{\theta-1}\left[m_{F,B} + \phi\, m_{I,B}\right]}_{\equiv u_B}, \tag{A.11}$$

so that at an interior equilibrium, $\tilde{\Delta}(\hat{K}) = \hat{K}$. An equilibrium is stable if $\tilde{\Delta}'(\hat{K}) < 1$. Consider an interior, stable, equilibrium. We prove that as L_A, λ_A, F_A rise, $\hat{K}$ rises. We make clear which results require the parameter assumption that, $\theta > (1-\theta)\left(1 + \frac{L_A}{F_A + F_B}\right)$. This is a more restrictive assumption that $\theta > 1/2$. We show the F_B comparative static is ambiguous and depends upon $\hat{K}$.

Preliminaries: conditions for $\frac{\partial u_A}{\partial m_{F,A}} > 0$. We have that

$$\frac{m_{I,A}}{m_{F,A}} = \frac{1}{\phi}\left(1 + \frac{L_A}{F_A + H(\hat{K})F_B}\right), \tag{A.12}$$

and

$$\log u_A = (\theta - 1)\log m_{F,A} + (\theta - 1)\log m_{I,A} + \log(m_{F,A} + \phi m_{I,A}). \tag{A.13}$$

Then

$$\frac{\partial \log u_A}{\partial m_{F,A}} = (\theta - 1)\frac{1}{m_{F,A}} + \frac{1}{m_{F,A} + \phi m_{I,A}}, \tag{A.14}$$

which is positive if:

$$m_{F,A} > (1 - \theta)(m_{F,A} + \phi m_{I,A}), \tag{A.15}$$

or

$$\theta > (1 - \theta)\left(1 + \frac{L_A}{F_A + H(\hat{K})F_B}\right). \tag{A.16}$$

We assume the parameter restriction that

$$\theta > (1 - \theta)\left(1 + \frac{L_A}{F_A + F_B}\right), \tag{A.17}$$

which ensures that $\frac{\partial u_A}{\partial m_{F,A}} > 0$.

Preliminaries: proof that $\frac{\partial u_A}{\partial m_{I,A}} > 0$. Next, we have that

$$\frac{\partial \log u_A}{\partial m_{I,A}} = (\theta - 1)\frac{1}{m_{I,A}} + \frac{\phi}{m_{F,A} + \phi m_{I,A}}, \tag{A.18}$$

which is positive if:

$$\phi m_{I,A} > (1 - \theta)(m_{F,A} + \phi m_{I,A}), \tag{A.19}$$

or

$$\theta\left(1 + \frac{L_A}{F_A + H(\hat{K})F_B}\right) > (1 - \theta), \tag{A.20}$$

which is always satisfied for $\theta > 1/2$.

Comparative statics for L_A. Given these results, it follows that $\frac{\partial u_A}{\partial L_A} > 0$. This because $m_{I,A}$ is increasing in L_A and u_A is increasing in $m_{I,A}$. Then, we have that

$$\frac{\partial \hat{K}}{\partial L_A} = -\frac{\partial(\tilde{\Delta}(\hat{K}) - \hat{K})}{\partial \hat{K}} / \frac{\partial u_A}{\partial L_A} \tag{A.21}$$

Since $\tilde{\Delta}'(K) - 1 < 0$ (stability) and $\frac{\partial u_A}{\partial L_A} > 0$, we have that $\frac{\partial \hat{K}}{\partial L_A} > 0$.

Comparative statics for λ_A. A similar argument follows for λ_A: u_A is increasing in λ_A and thus

$\frac{\partial \hat{K}}{\partial \lambda_A} > 0$.

Comparative statics for F_A. For F_A, note that $m_{F,A}$ and $m_{I,A}$ are increasing in F_A. Further, U_A is increasing in $m_{F,A}$ and $m_{I,A}$ under the more stringent restriction on θ. Thus it follows that $\frac{\partial \hat{K}}{\partial F_A} > 0$

Comparative statics for F_B. Last, consider F_B. All of the masses are increasing in F_B, and both u_A and u_B are increasing in the masses. Thus the sign of the comparative static depends on which effect dominates, and we get the ambiguous result. As $\hat{K}$ goes to ∞ and $H(\hat{K})$ goes to one, the effect on u_A will dominate and hence increases in F_A leads to a rise in $\hat{K}$.

□

Proof of Proposition 4 (Asset pricing). The first part of the proposition requires us to prove that

$$\frac{\partial (P_{0,A} - \beta^2)}{\partial \hat{K}} > 0. \tag{A.22}$$

Start by noting that

$$\log(P_{0,A} - \beta^2) = \log \lambda_A + \theta \log m_{F,A} + (\theta - 1) \log m_{I,A} + \text{constant}. \tag{A.23}$$

We differentiate this with respect to $H(\hat{K})$:

$$\frac{\partial \log(P_{0,A} - \beta^2)}{\partial H} = \theta \frac{\partial \log m_{F,A}}{\partial H} + (\theta - 1) \frac{\partial \log m_{I,A}}{\partial H} \tag{A.24}$$

$$= F_B \left(\frac{\phi \theta}{m_{F,A}} + \frac{\theta - 1}{m_{I,A}} \right) \tag{A.25}$$

This is positive as long as

$$\frac{\theta}{1 - \theta} > \frac{F_A + H(\hat{K}) F_B}{L_A + F_A + H(\hat{K}) F_B}, \tag{A.26}$$

which always holds given that $\theta > \frac{1}{2}$. This implies that $\frac{\partial \log(P_{0,A} - \beta^2)}{\partial H} > 0$. Since $H(\hat{K})$ is a monotone increasing function of $\hat{K}$, we then obtain the desired result in equation (A.22). The second part of the proposition follows immediately from (A.23) given that $\theta - 1 \leq 0$.

□

Proof of Proposition 5 (Cross-sectional complementarities). Define

$$\tilde{\Delta}^{+}(\hat{K}^{+}) = \lambda_A m_{F,A}^{\theta} m_{I,A}^{\theta - 1} - \lambda_B m_{F,B}^{\theta} m_{I,B}^{\theta - 1}, \tag{A.27}$$

$$\tilde{\Delta}^{-}(\hat{K}^{-}) = \lambda_A \phi m_{F,A}^{\theta - 1} m_{I,A}^{\theta} - \lambda_B \phi m_{F,B}^{\theta - 1} m_{I,B}^{\theta}. \tag{A.28}$$

where now the masses are given by

$$m_{F,A} = \phi \left[F_A + H(\hat{K}^{-}) F_B^{-} \right], \qquad m_{F,B} = \phi \left[1 - H(\hat{K}^{-}) \right] F_B^{-}, \tag{A.29}$$

and

$$m_{I,A} = F_A + L_A + H(\hat{K}^{+}) F_B^{+}, \qquad m_{I,B} = L_B + \left[1 - H(\hat{K}^{+}) \right] F_B^{+}. \tag{A.30}$$

Consider an interior equilibrium. The expression (A.27), holding $\hat{K}^+$ fixed, is strictly increasing in $\hat{K}^-$ in an interior equilibrium, and weakly increasing in an equilibrium with no foreign denomination. Conversely, holding $\hat{K}^-$ fixed, expression (A.28) is strictly increasing in $\hat{K}^+$ in an interior equilibrium, and otherwise weakly increasing. This yields the desired result. □

Proof of Proposition 3 (Sovereign incentives). We start from the sovereign's objective function, which for a given country j has the form

$$W_j = L_j C_j + F_j \mathcal{U}_j(L_j), \tag{A.31}$$

where C_j is the convenience yield and $\mathcal{U}_j$ is average utility for j-firms. We take existing debt to be inframarginal, in the sense of earning the convenience yield set at issuance: therefore, for an infinitesimal perturbation, C_j in the expression above does not depend on L_j (i.e., $\frac{\partial C_j}{\partial L_j}$). We want to show that the leader country in a dominance equilibrium (A) has a stronger incentive to increase sovereign debt issuance L_j than the follower country (B), meaning that

$$\frac{\partial W_A}{\partial L_A} > \frac{\partial W_B}{\partial L_B}. \tag{A.32}$$

We begin by proving this result in the limit $H(\hat{K}) \to 1$. We start by noting that one can express the firm utility gradients as the sum of two components. A first component holds $\hat{K}$ constant, while the second component captures the additional effect due to firm foreign denomination:

$$\frac{\partial F_j \mathcal{U}_j}{\partial L_j} = \nabla^{\text{No FD}}_{F_j \mathcal{U}_j} + \nabla^{\text{FD terms}}_{F_j \mathcal{U}_j}. \tag{A.33}$$

The relative scale of these two components depends on the foreign denomination gradient $\frac{\partial H(\hat{K})}{\partial L_j}$. The component $\nabla^{\text{No FD}}_{F_j \mathcal{U}_j}$ is multiplied by a scaling factor of one, while the component $\nabla^{\text{FD terms}}_{F_j \mathcal{U}_j}$ has scale $F_B \frac{\partial H(\hat{K})}{\partial L_j}$.

It is clear then that the relative scale of the two terms depends on the level of foreign denomination $H(\hat{K})$. We have that:

$$F_B \frac{\partial H(\hat{K})}{\partial L_j} = F_B h(\hat{K}) \frac{\partial \hat{K}}{\partial L_j}. \tag{A.34}$$

As long as the distribution $H(\cdot)$ has a finite mean (which the Pareto form used in our parametric analyses does, for $\alpha > 1$), this tends to zero in the limit $H \to 1$ (or equivalently, $\hat{K} \to \infty$):[2]

$$\lim_{H \to 1} F_B \frac{\partial H(\hat{K})}{\partial L_j} = 0. \tag{A.35}$$

Therefore for large H, the component $\nabla^{\text{FD terms}}_{F_j \mathcal{U}_j}$ becomes negligible relative to the component $\nabla^{\text{No FD}}_{F_j \mathcal{U}_j}$,

[2] To see this, consider that $\lim_{\hat{K} \to \infty} \frac{\partial}{\partial \hat{K}} \int_{\underline{K}}^{\hat{K}} K H'(K) \mathrm{d}K = 0$.

and we can use the following approximation:

$$\frac{\partial F_j \mathcal{U}_j}{\partial L_j} \approx \nabla^{\text{No FD}}_{F_j \mathcal{U}_j}, \tag{A.36}$$

which becomes exact as $H \to 1$.

Allowing for marginal pricing on corporate debt issuances, and using the approximation above, we then have that

$$\frac{\partial F_A \mathcal{U}_A}{\partial L_A} = \phi \lambda_A F_A m_{F,A}^{\theta-1} m_{I,A}^{\theta-2} \left[\theta L_A + (2\theta - 1)(F_A + H(\hat{K}) F_B)\right] > 0, \tag{A.37}$$

where the inequality follows from the fact that $\theta > \frac{1}{2}$. We also have that

$$\begin{aligned} \frac{\partial F_B \mathcal{U}_B}{\partial L_B} = \lambda_B \left(L_B + (1 - H(\hat{K})) F_B\right)^{\theta-2} \left(\phi F_B (1 - H(\hat{K}))\right)^{\theta} \Big[\theta L_B \\ + F_B (H(\hat{K}) - 1 + 2\theta - 2\theta H(\hat{K}))\Big] - \frac{\partial \mathcal{C}(\hat{K})}{\partial L_B}, \end{aligned} \tag{A.38}$$

where $\mathcal{C}(\hat{K})$ summarizes the foreign currency issuance costs paid by firms. In the limit $H \to 1$, these become:

$$\lim_{H\to 1} \frac{\partial F_A \mathcal{U}_A}{\partial L_A} = \phi \lambda_A F_A m_{F,A}^{\theta-1} m_{I,A}^{\theta-2} \left[\theta L_A + (2\theta - 1)(F_A + F_B)\right] > 0, \tag{A.39}$$

$$\lim_{H\to 1} \frac{\partial F_B \mathcal{U}_B}{\partial L_B} = - \lim_{H\to 1} \frac{\partial \mathcal{C}(\hat{K})}{\partial L_B}, \tag{A.40}$$

Next, note that

$$\frac{\partial \mathcal{C}(\hat{K})}{\partial L_B} = \frac{\partial}{\partial L_B} \int_{\underline{K}}^{\hat{K}} K H'(K) \mathrm{d}K = H'(K) \frac{\partial \hat{K}}{\partial L_B}, \tag{A.41}$$

which tends to zero for $H \to 1$ for distributions $H(\cdot)$ with a finite mean:

$$\lim_{H\to 1} \frac{\partial \mathcal{C}(\hat{K})}{\partial L_B} = 0. \tag{A.42}$$

It follows that

$$\lim_{H\to 1} \frac{\partial F_B \mathcal{U}_B}{\partial L_B} = 0. \tag{A.43}$$

We can then conclude that, in the limit,

$$\frac{\partial F_A \mathcal{U}_A}{\partial L_A} > \frac{\partial F_A \mathcal{U}_B}{\partial L_B}. \tag{A.44}$$

The gradient of the seignorage component of the sovereign's objective is simply the convenience yield itself, independently of $H(\hat{K})$, given inframarginal pricing of the existing debt:

$$\frac{\partial L_j C_j}{\partial L_j} = C_j. \tag{A.45}$$

Therefore as long as $C_A > C_B$, we obtain that $\frac{\partial W_A}{\partial L_A} > \frac{\partial W_B}{\partial L_B}$ in the limit $H \to 1$ as desired.

These limiting results imply that for any real value $\varepsilon > 0$, there exists a value $H^\dagger = H(\hat{K}^\dagger)$ such that for all $H > H^\dagger$, and hence for all $\hat{K} > \hat{K}^\dagger$, we obtain that

$$| \nabla^{\text{FD terms}}_{F_j \mathcal{U}_j} | < \varepsilon \quad \forall j, \tag{A.46}$$

and

$$\nabla^{\text{No FD}}_{F_A \mathcal{U}_A} - \nabla^{\text{No FD}}_{F_B \mathcal{U}_B} + \varepsilon > 0. \tag{A.47}$$

By setting ε arbitrarily small, we obtain the desired result in the proposition. □

Proof of Proposition 6 (Normative implications). Global welfare is $\mathcal{W} = W_A + W_B$. Call K^* the global welfare optimizing K. We wish to show that $K^* > \hat{K}_{\max}$ in the case $\theta = 1$. It is sufficient to show that,

$$\left.\frac{\partial \mathcal{W}}{\partial K}\right|_{K=\hat{K}} > 0. \tag{A.48}$$

We have,

$$\frac{\partial W_A}{\partial K} = L_A \lambda_A (\phi F_B h) + F_A \lambda_A 2(\phi F_B h), \tag{A.49}$$

and,

$$\begin{aligned} \frac{\partial W_B}{\partial K} \quad = \quad & -L_B \lambda_B (\phi F_B h) - (1-H) F_B \lambda_B 2(\phi F_B h) \\ & -\lambda_B (m_{F,B} + \phi m_{I,B}) h F_B + \lambda_A (m_{F,A} + \phi m_{I,A}) h F_B - \hat{K} h F_B \\ & + F_B H \lambda_A 2(\phi F_B h). \end{aligned}$$

Notice that the middle line here is equal to zero: it is the equilibrium condition that determines $\hat{K}$. Then,

$$\left.\frac{\partial \mathcal{W}}{\partial K}\right|_{K=\hat{K}} = \{L_A \lambda_A - L_B \lambda_B + 2F_A \lambda_A - 2F_B \lambda_B + 2F_B H \lambda_A + 2F_B H \lambda_B\} (\phi F_B h), \tag{A.50}$$

which is positive in the case of A dominance ($L_A > L_B, \lambda_A \geq \lambda_B$).[3] □

A.4 Derivations for the Model With Aggregate Risk

This section provides additional derivations for the extended version of the model with aggregate risk that we introduce in Section 5.5. We analyze this extended version of the model in the $\theta = 1$ case. To start, we consider the equilibrium at time t_1 in state ω. The masses of liquidity providers are

$$m^{\omega}_{I,j} = L^{\omega}_j + F_j \tag{A.51}$$

[3] We also need $\mathcal{W}$ to be globally concave for this approach to be valid. It is straightforward to differentiate $\frac{\partial \mathcal{W}}{\partial K}$ and show that the second derivative is negative as long as $h' < 0$, which the Pareto distribution satisfies.

where we recall that we allow for state-contingency in the supply of government assets L^A_ω, but we assume that F_j is not state-contingent. Correspondingly, the masses of liquidity demanders are

$$m^\omega_{F,j} = \phi^\omega F_j. \tag{A.52}$$

The two-sided match probabilities are then

$$\underbrace{\alpha^\omega_{F,j} = \lambda_j m^\omega_{I,j}}_{P(\text{Buyer finds a seller})} \quad , \qquad \underbrace{\alpha^\omega_{I,j} = \lambda_j m^\omega_{F,j}}_{P(\text{Seller finds a buyer}).} \tag{A.53}$$

The surplus from are match remains $1-\beta$, independent of the aggregate state. The date t_0 price of the private bond is therefore

$$P_{0,j} = E\left[\underbrace{\alpha^\omega_{I,j}\beta\left[\beta + (1-\eta)(1-\beta)\right]}_{P(\text{Matched})\times\text{PV of Profit}} \quad + \quad \underbrace{(1-\alpha^\omega_{I,j})\beta^2}_{P(\text{Not Matched})\times\text{PV of 1}}\right], \tag{A.54}$$

which we rewrite as

$$P_{0,j} = \beta^2 + (1-\eta)\lambda_j E[m^\omega_{F,j}]\beta(1-\beta). \tag{A.55}$$

Firm utility at date t_0 is

$$u^F_{i,j} = P_{0,j} + \beta E[\phi^\omega \alpha^\omega_{F,j}]\eta(1-\beta), \tag{A.56}$$

which, substituting in for $P_{0,j}$, becomes

$$u^F_{i,j} = \beta^2 + (1-\eta)\lambda_j E[m^\omega_{F,j}]\beta(1-\beta) + \beta\lambda_j E[\phi^\omega m^\omega_{I,j}]\eta(1-\beta). \tag{A.57}$$

As before, we take the case $\eta = \frac{1}{2}$ and rewrite:

$$u^F_{i,j} - \beta^2 = \frac{1}{2}\lambda_j\beta(1-\beta)\Bigg(E[m^\omega_{F,j} + \phi^\omega m^\omega_{I,j}]\Bigg). \tag{A.58}$$

This expression is similar to that of the non-stochastic case, except that the masses are now stochastic.

We next substitute in for the masses and rewrite:

$$\begin{aligned} u^F_{i,j} - \beta^2 &= \frac{1}{2}\lambda_j\beta(1-\beta)\Bigg(E[\phi^\omega(2F_j + L^\omega_j)]\Bigg) && \text{(A.59)}\\ &= \frac{1}{2}\lambda_j\beta(1-\beta)\Bigg(E[\phi^\omega](2F_j + E[L^\omega_j]) + cov[\phi^\omega, L^\omega_j]\Bigg) && \text{(A.60)}\end{aligned}$$

This yields the indifference condition presented in the main text:

$$\lambda_A\Bigg(\mathbb{E}[\phi_\omega](2(F_A + H(\hat{K})F_B) + \mathbb{E}[L^A_\omega]) + \mathrm{Cov}[\phi_\omega, L^A_\omega]\Bigg) - \hat{K} = \lambda_B\mathbb{E}[\phi_\omega]\Bigg(2(1-H(\hat{K}))F_B + L_B\Bigg). \tag{A.61}$$

A.5 Investments in Private Revenue Pledgeability

We consider an extension of the model in which the sovereigns can invest in country-specific financial innovation which improves the pledgeability of firms' revenue streams. We do this is the case where $\theta = 1$. In this extended version of the model, when firms apply for a bond issuance, they find out whether their project revenues are fully pledgeable (probability ρ_j) or non-pledgeable (probability $1 - \rho_j$). The borrowing ability is idiosyncratic, so that *ex ante* a given firm is able to pledge its revenues with probability ρ_j, and the law of large numbers applies across firms. After firms decide the currency in which to issue their debt, they discover their pledgeability, and if they choose to borrow they incur the fixed cost K_i, as in the basic model. Thus, ρ_j captures the pledgeability of firm revenues in country j. We set $\theta = 1$ for illustrative purposes.

The expected utility of borrowing in country j is then proportional to

$$\rho_j \lambda_j \left[m_{F,j} + \phi \, m_{I,j} \right], \tag{A.62}$$

and the equilibrium condition for the marginal firm is now:

$$\rho_A \left[\lambda_A (m_{F,A} + \phi \, m_{I,A}) - \hat{K} \right] = \rho_B \left[\lambda_B (m_{F,B} + \phi \, m_{I,B}) \right]. \tag{A.63}$$

Increasing ρ_A increases the benefits of issuing in currency A in equation (A.63), thereby requiring the equilibrium $\hat{K}$ to adjust to a higher value. As in the case with the government's commitment technology, the incentive to invest is large in the dominant country, and moreover these stronger incentives are further reinforced by the endogenous increase in foreign denomination caused by the investments in private pledgeability:

$$\frac{\partial W_A}{\partial \rho_A} > \frac{\partial W_B}{\partial \rho_B}, \quad \frac{\partial^2 W_A}{\partial \rho_A \partial \hat{K}} > 0, \quad \frac{\partial \hat{K}}{\partial \rho_A} > 0. \tag{A.64}$$

Proof. The equilibrium masses of firms are:

$$m_{F,A} = \rho_A \phi \left[F_A + H(\hat{K}) F_B \right], \qquad m_{F,B} = \rho_B \phi \left[1 - H(\hat{K}) \right] F_B, \tag{A.65}$$

while the masses of liquidity suppliers are,

$$m_{I,A} = L_A + \rho_A F_A + \rho_A H(\hat{K}) F_B, \qquad m_{I,B} = L_B + \rho_B \left[1 - H(\hat{K}) \right] F_B. \tag{A.66}$$

Consider the objectives for country A and B. We have that

$$\begin{aligned} W_A &= L_A(\lambda_A m_{F,A}) + \rho_A F_A \lambda_A (m_{F,A} + \phi m_{I,A}) \\ &= L_A \lambda_A \rho_A \phi \left[F_A + H(\hat{K}) F_B \right] + \rho_A F_A \lambda_A \Big(\rho_A \phi \left[F_A + H(\hat{K}) F_B \right] + \phi (L_A + \rho_A F_A + \rho_A H(\hat{K}) F_B) \Big) \end{aligned}$$

and,

$$\begin{aligned}
W_B &= L_B \lambda_B m_{F,B} + \rho_B F_B (1 - H(\hat{K})) \lambda_B (m_{F,B} + \phi m_{I,B}) + U_{B \to A} \\
&= L_B \lambda_B \rho_B \phi \left(1 - H(\hat{K})\right) F_B \\
&\quad + \rho_B F_B (1 - H(\hat{K})) \lambda_B \left(\rho_B \phi \left(1 - H(\hat{K})\right) F_B + \phi(L_B + \rho_B \left(1 - H(\hat{K})\right) F_B) \right) + U_{B \to A}
\end{aligned}$$

It is straightforward to see that:

- $\frac{\partial W_A}{\partial \rho_A} > \frac{\partial W_B}{\partial \rho_B}$ since $L_A > L_B$, $\lambda_A \geq \lambda_B$ and $F_A \geq F_B$.
- $\frac{\partial^2 W_A}{\partial \rho_A \partial \hat{K}} > 0$ since $\frac{\partial m_{F,A}}{\partial \hat{K}}$ and $\frac{\partial m_{I,A}}{\partial \hat{K}}$ are positive and $\frac{\partial \hat{K}}{\partial \rho_A}$ is positive.

$\square$

B Further Historical Details

This section provides additional details which supplement our discussion of the various historical episodes.

B.1 Bank of Amsterdam

The Dutch florin created by the Bank of Amsterdam in 1609 was the first global currency. For much of history, transactions and debts around the world were primarily settled in metallic coins. However, hundreds of domestic and foreign varieties existed, and using them entailed large transaction costs such as transportation, insurance, and assayance.[4] The difficulty of enforcing quality created incentives to debase the currency and reduce the circulating supply of high-quality coins. These costs compounded the difficulty of coordinating on the coins that were valid for settling a debt. While negotiable credit instruments such as the bill of exchange reduced the need to transfer coins, they still required a unit of denomination and so settlement ultimately relied on an uncertain supply of physical assets.

The Bank of Amsterdam was chartered by the City of Amsterdam to provide a high quality standardized currency that would reduce settlement frictions. The Bank primarily did so by creating a currency that existed on its ledgers ("bank florin") that was backed by coin and could be transferred across accounts freely. The City of Amsterdam initialized the pool of florin available for settlement by requiring that all large bills of exchange drawn and/or payable in Amsterdam had to be settled at the Bank, i.e., denominated in florin.[5] Relative to the uncertain supply of specific metal coins in circulation anywhere else in the world, the Bank provided florin the advantage that there would be a ready supply for payments in Amsterdam.

Bank accounts were freely provided to anyone, and florins were credited to accounts for deposits of recognized coins. These coins backed the florins that could be withdrawn as current guilders in a narrow bank model. Since the Bank charged a fee for withdrawals, it was usually less costly to trade florin for current guilder in a secondary "open market."[6] In that market, the *agio* was the market exchange rate between the bank florin and the current guilder.

Rotterdam, a neighboring mercantile city, also created its own exchange bank modeled after the Bank of Amsterdam. While the two institutions maintained separate balance sheets, Rotterdam adopted Amsterdam's *agio* because merchants preferred florin (Van der Borght, 1896, p. 209). Rotterdam provided a system where all deposits and withdrawals of guilders were made allowing for the Amsterdam *agio* and thus used Amsterdam's florin as the unit of account (Carey, 1818, p. 369). Rotterdam also conceded to provide current accounts, which were the primary means for merchants

[4] An ordinance in the Dutch Republic from 1606 officially recognized 25 gold and 14 silver trade coins from 35 domestic mints, but many more varieties circulated, and the Republic officially published exchange rates for almost 1000 coins (Roberds and Velde, 2016, p. 344).

[5] The first ordinance in February 1609 applied to bills over 600 guilders; in 1642 year this was revised to include bills over 300 guilders. As a result, all merchants kept an account at the Bank, and the Bank maintained two to three thousand accounts at any given time (Van Dillen, 1934, p. 107).

[6] The withdrawal fee of 1.5% covered the costs that the Bank of Amsterdam incurred to mint current guilders for deposits of inferior coins (judged by their metallic content).

to access florin by way of guilders and much more heavily used than its own bank money (Van der Borght, 1896, p. 210). In addition, the Bank of Rotterdam, despite requiring large bills of exchange to be settled in its own bank florin, also settled bills payable in florin in Amsterdam. In these ways, Rotterdam provided access to florins to the extent possible given its separate balance sheet.

In 1683, the Bank also introduced a *receipts* technology that operated like a modern day repurchase agreement. The Bank of Amsterdam advanced florin for short-term deposits of specie and metal bars. Depositors were issued a receipt, negotiable and renewable with an initial maturity of six months, for the right to withdraw the specific metal they deposited.[7] This technology broadened the set of assets that could be converted into florin beyond the original set of trade coins, and it was beneficial for both the Bank of Amsterdam and for private parties. The former gained from the metal deposits, which became part of the Bank's assets if the receipt expired, and the latter was able to obtain florin for settlement without needing to convert them into eligible trade coins for deposit at market value.[8]

Following the introduction of the repo facility, the quantity of florins at the Bank of Amsterdam doubled from approximately eight million to sixteen million from the mid 17th to the beginning of the 18th century (Quinn and Roberds, 2014b). These complementary innovation forces are also at play in the rise of Amsterdam. For instance, the receipts technology (i.e., repurchase facility) that created florin claims out of raw specie was only introduced in 1683, almost seven decades after the Bank's establishment. The fact that florin balances doubled after its introduction is indicative of the force identifed in our model that increases in λ_j generate further foreign denomination (Quinn and Roberds, 2014b).

After a century of dominance, the Bank of Amsterdam eventually became a victim of its own success. Intermediation in bank florins was profitable for the Bank, and it routinely turned over its wealth to the City of Amsterdam, leaving it with little capital buffer. It also made advances to the Dutch East India Company (VOC), which eventually led to runs on bank florin after the VOC came close to failure following the fourth Anglo-Dutch War in 1784. The French invasion in 1795 led to a drop in the *agio* to -14%, after which it never fully recovered, and eventually the Bank was formally dissolved in 1819.

B.1.1 Dutch versus Spanish Trade

The figures on Spanish trade per capita and the population are taken from Ortiz-Ospina et al. (2018) and Allen (2003) on p. 438 (Table A1) respectively. Data are available for four years (1600, 1700,

[7] There was a large secondary market in receipts. Receipts could be redenominated in smaller face values, and they were renewable by paying the withdrawal fees. Withdrawal fees with receipts (0.125% for silver and 0.25% for gold) were much lower than that for current guilders (1.5%) because the Bank did not need to mint guilders to meet withdrawal demands. Around this time it appears the Bank of Amsterdam eliminated the right to withdraw from its accounts, which has led some authors to argue that the florin was an early fiat currency (Quinn and Roberds, 2014b).

[8] Given the wide variety of specie circulating, the demand and supply for specific coins varied significantly, and market prices were usually in flux. The receipts technology made it possible to transact on the Bank's mandated value for the specie while retaining the ability to withdraw and sell at a future date when prices rose. It also supported a large trade in precious metals since the freely-traded receipts were equivalent to advances on pledges of the underlying metals.

1750 and 1800). Dutch trade comes from Zanden and Leeuwen (2018). We extract the four data points to compute Spanish trade relative to Dutch trade in the $17^{th} - 19^{th}$ centuries. Pound-guilder exchange rates are taken from Denzel (2017) (Figure 28.1).

Table B.1: Raw data for comparing Dutch and Spanish trade

Year	Trade Holland (in million guilder)	Trade Spain (in £ per capita)	Population Spain	Exchange rate £-guilder
1600	13.462	.18907407	8,700,000	0.095
1700	12.152	.31009123	8,600,000	0.1
1750	10.221	.57871836	9,600,000	0.095
1800	16.241	.48912659	13,000,000	0.09

In order to create a direct comparison, we convert the Spanish trade in £ per capita to country-level trade in guilders.

Table B.2: Trade volumes (in millions of guilders)

Year	Holland	Spain
1600	13.462	17.315
1700	12.152	26.667
1750	10.221	58.481
1800	16.241	70.651

B.1.2 The Italian City-States Experience

The equilibrium of Dutch florin dominance emerged out of a broader historical context in which, prior to the Dutch era in the 17th and 18th centuries, the Italian city-states of the Renaissance were prominent financial centers. Yet the Republics of Genoa, Venice, and Florence conducted their finance in a constellation of local currencies, such as the Genoese and Venetian lira and the Florentine florin, and none achieved the centrality that the Dutch florin would attain in subsequent centuries.

Our model rationalizes this difference between the Italian city-states and the later Dutch experience and attributes it to the economic mechanisms centered on increasing returns to scale in liquidity provision. The multipolar equilibrium of the several Italian currencies of roughly equal importance is stable as long as these currencies have separate and approximately symmetric underlying liquidity pools (as determined by L_j).

We consider this to be an accurate representation of the historical context in which the constellation of Italian currencies circulated in the form of physical coins and therefore faced high settlement frictions. While these city-states had large banks (including the Bank of San Giorgio dating from 1407 in Genoa and the Medici Bank from 1397 in Florence), none of them invested in creating a large and steady supply of safe debt in a common currency (L_j) as Amsterdam did.[9] The Genoese

[9]Unlike Amsterdam, the Italian banks were slow to adopt payments by account transfers. For instance, it was not until 1675 that the Bank of San Giorgio in Genoa issued depositors transferable vouchers reflecting deposit accounts (Willis, 1943, p. 12).

lira and the Florentine florin therefore remained unremarkable in terms of their liquidity properties like the other coins of the era, and this system corresponds to equilibrium point 0 of Figure 2.

B.2 Great Britain

This initial point established the pound as a dominant currency with substantial foreign denomination from foreign sovereigns.[10] The institutional developments in the legal structure of bills of exchange and the role of the Bank of England also reflect investments in overall market liquidity λ_A. These investments continued throughout the 19th century as dominance engendered foreign denomination, liquidity, and increased incentives to innovate.

The Bank of England was a key institution in the London money market, founded in 1694 as a note-issuing private corporation that was granted several privileges in return for raising and administering the Crown's debt.[11] During the early part of its history, the Bank competed with other private banks to increase its note circulation and raise its profits.[12] In this respect, the Bank was like any other private firm that was incentivized to issue safe debt in order to benefit from the yield premium in equation (A.55). It was very successful in establishing a sound reputation for its notes, and by the late 18th century, Bank notes became synonymous with the pound sterling (Thornton, 2017).[13]

In 1833 and subsequently 1844, the *de facto* equivalence between the pound and Bank notes became *de jure* with passage of the Bank Notes Act and the Bank Charter Act respectively. The former made Bank notes legal tender while the latter consolidated the entire note issuance onto the Bank of England's balance sheet where it was fully backed in gold reserves above the allowed fiduciary issue.[14] The full note circulation of the Bank of England therefore officially contributed to the supply of G_A following the 1840s.

A second important innovation was the legal codification of the contractual terms for bills of exchange, which coordinated the market on the terms of borrowing and the procedures for default. Bills of exchange were the primary London money market instrument, and each time one was traded

[10]The vast majority of the sovereign debt issued in London after the Napoleonic Wars were by Latin American and other European nations rather than the British colonies (Meyer et al., 2022).

[11]Like most currencies during this era, the pound sterling referred to specific metallic coins, and obligations denominated in sterling were contracted to be repaid in those coins. However, coins were inconvenient for the reasons already discussed, and private banks found it profitable to issue paper notes denominated in sterling (i.e., claims on sterling coin). The privileges restricted banking competition and gave the Bank of England a monopoly over note issuance. From 1697 until 1844, only the Bank of England could raise equity; all other banks were restricted to partnerships of six or fewer (after 1844, this was altered to a radius of 25 miles around London). In 1708, the Bank was granted an exemption to laws restricting bank note issuances to private partnerships (Broz and Grossman, 2004).

[12]The Banking Act of 1826 required the Treasury to monitor the amount of small-denomination notes issued by the Bank following the 1825 crisis in which the Bank was seen to have abused its monopoly (Scammell, 1968, p. 132).

[13]Estimates of historical convenience yields in the era of British pound dominance include Chen et al. (2022) and Payne et al. (2022).

[14]The limits of the Bank's note supply was therefore primarily governed by the gold reserves at the bank and secondarily by the government-determined fiduciary issue. Private bank notes already in circulation were allowed to remain, but no new notes could be issued, and banks lost their right when they merged. The Bank Charter Act could be suspended during financial crises when there was large demand for Bank notes.

("discounted"), the seller guaranteed ("endorsed") the bill. These endorsements were legally equivalent to being the original borrower, and so each endorser was equally liable. The generality of these conditions were constantly tried at court and established a strong legal precedent.[15] These laws reduced the information sensitivity of bills and collectively raised the safety and liquidity of all bills of exchange with multiple endorsers, regardless of the idiosyncratic characteristics of the ultimate borrower. Thus, these innovations made private debt money-like in the sense of Dang et al. (2017).[16]

A third notable institution was the Bank of England's role as a credible and reliable lender of last resort to the financial sector. During the banking crises of 1847, 1857, and 1866, the Bank obtained permission from the Treasury to suspend the Bank Charter Act in order to meet all demand for Bank notes.[17] In fact, the Bank's behavior during the crisis of 1866 was the basis for Bagehot's rules for central banking (Bagehot, 1873). As a lender of last resort, the Bank provided liquidity at its discount window by converting private bills of exchange into pounds sterling, thereby *de facto* became a backstop to the private bills market. This backstop officially only applied to high-quality bills—those first guaranteed ("accepted") by large merchant banks that held accounts at the Bank of England—but like all liquidity backstops, its existence reduced the occurrence of market freezes and increased the willingness of private firms to lend in all states.[18] These forces together led issuers to prioritize denominating issues in sterling, thereby increasing the quantity of safe pound-denominated debt in the London money market.

The Bank of England acting as a lender of last resort was a major transition from its earlier history in the 18th century when discounting and note issuance was a profit-maximizing endeavor. At that point, the Bank's discount window followed the market and became similarly unavailable during downturns and crises.[19] As the London money market deepened and the pound sterling gained dominance in the 19th century, the Bank increasingly took on a more formal role within

[15] A Parliametary report from 1837 describes the legal protections against default: "a holder of a bill of exchange can bring actions at one and the same time, against every party whose name is attached to it, and in the event of the failure of them all, can prove upon the estate of each for the full value of the bill" (Joplin, 1837, p. 17).

[16] An additional factor is that the Act of 1833 exempted short-term bills of exchange from the Usury Laws, which also expanded the market's general willingness to hold them (Scammell, 1968).

[17] Since the Bank Charter Act limited the supply of Bank notes to the Bank's gold reserve, suspending it (and therefore the gold standard) was the only way to ensure they could meet demand. Even when the gold reserve was high, the presence of a limit reduced liquidity in the market. It is worth noting that obtaining permission to suspend the Bank Charter Act was sufficient, and Great Britain did not actually suspend the gold standard during this period.

[18] The high quality bills eligible at the Bank of England became a class of their own, and the financial press throughout this period reported the rates on "Bank" bills separately from "trade" bills (Xu, 2022).

[19] Scammell writes, "All in all the discounting of bills by the Bank in the early 19th century must be seen primarily as a prosperous business of the Bank and only very secondarily as a manifestation of credit policy," (Scammell, 1968, p. 144). For example, during the 1797 crisis, Parliament assumed the role of being a liquidity provider by issuing exchequer (treasury) bills to the market (Thornton, 2017, p. 98). The subsequent crisis in 1825 provides a microcosm into the transition that took place. Early in the year, the Bank of England closed its discount window because it anticipated a financial market downturn. This action in itself "created an atmosphere of misgiving and potential crisis," (Scammell, 1968, p. 131). However, when the crisis peaked in November with numerous failures in London, the Bank reversed its earlier decision and made discounts and advances on government securities and private bills. Thereafter starting in 1830, it allowed bill brokers to access the discount window for the first time, after recognizing that these institutions were important conduits of liquidity.

the government. This was despite the fact that it remained privately owned by stockholders until 1944 and run by Governors and Courts of Directors that primarily stemmed from the merchant and banking classes (Cassis, 1994, p. 85). The Bank's transitioning role in the money market given its dual identities reflects how the benefits of agglomeration accrued to both the government and to the private sector, all embodied in a single institution.

One final development during this period that contributed to maintaining the dominant equilibrium is the growth of international banking, which facilitated access to the pound sterling in locations around the world. British overseas banking institutions generally followed the business model of issuing deposits and shares domestically while lending via bills of exchange payable in London in their branches abroad. As in Amsterdam, the short term commercial bill became the dominant credit instrument internationally, with payments settled in London even for transactions that did not involve Great Britain.[20] The network of British banks increased the likelihood that foreign firms could hold pound obligations (or equivalently receive part of their profits in pounds), which in the context of the model we view as equivalent to reducing the cost of foreign currency issuance K_i, whether through a reduction of underlying FX exposures or via a reduction in the fixed cost of debt underwriting. Reducing this cost increases the mass of firms for which issuing in the foreign currency is profitable, as shown in equation (A.1).[21]

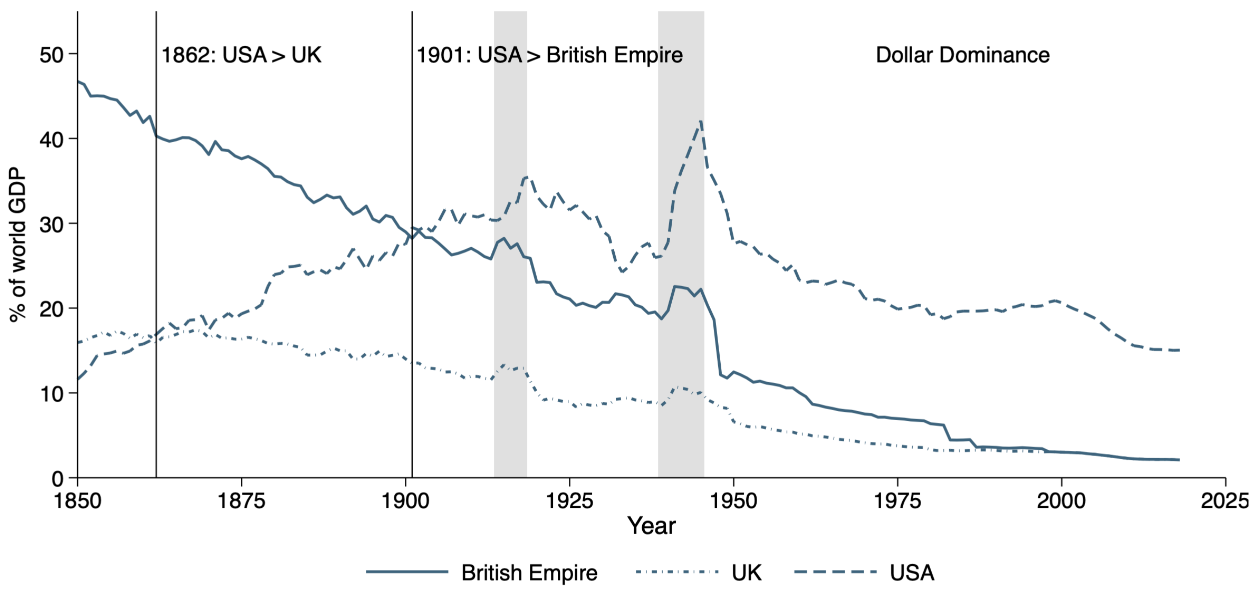
B.3 Comparing British Empire and US GDP

Figure B.2 plots the US, UK, and British Empire shares of global GDP from 1850 until today. This figure shows that the US overtakes UK GDP in the 1860s, and that the US overtakes the British Empire in 1901. These transitions in size occurred before the very beginning of the period of dollar dominance, which can be dated to the end of WWI at the very earliest, but is generally dated to the end of WWII.

[20]For example, "the bill on London enabled the banks [...] to finance a large share of international trade regardless of whether that trade touched Britain's shores," (Orbell, 2017, p. 8), and "wines from France, coffee from Brazil, sugar from the West Indies, and silk from Hong Kong were paid alike with bills on London," (Jenks, 1927, p. 69).

[21]Incidentally, both the French and the Germans followed the British model, often with explicit reference to expanding their currencies abroad. For instance, Edward Hurley, in his arguments for the creation for the US Federal Reserve System wrote, "The logical ambition of the German commercial policy is naturally to enthrone the Mark in the estimation of the world until it need pay no deference to the pound sterling."

Figure B.2: **Share of Global GDP**



Notes: We construct Figure B.2 using data from the Maddison Project Database (version 2020). Countries are counted as part of the British Empire using contemporary definitions. The UK includes Great Britain, Scotland, Wales, and Ireland. We include countries when they provide uninterrupted annual GDP data. The only country with interpolated data is India prior to 1884, in which missing GDP values were filled in based on the previously reported year. This interpolation method for India, chosen for illustrative reasons, avoids a significant spike in the Empire's GDP between 1883 and 1884 when India consistently enters our sample. This approach does not affect the year the USA took over the British empire, which is 1901.

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