Abstract

The international monetary system of the last four centuries has experienced the rise, persistence, and fall of specific currencies as the dominant unit of denomination in global debt contracts. We provide a liquidity-based theory to explain this pattern. Firms issue debt that can be extinguished by trading their revenues for financial assets of the same denomination. When asset markets differ in their liquidity, as modeled via endogenous search frictions, firms optimally choose to denominate their debt in the unit of the asset that is most liquid. Equilibria with a single dominant currency emerge from a positive feedback cycle, whereby issuing in the more liquid denomination endogenously raises the benefits of that denomination. This feedback mechanism has historically been seeded by governments that created the largest pool of liquid assets in the same denomination. Once dominance is established, a country’s costs of investing in the ability to create liquid assets, such as by increasing fiscal capacity, are lower, while the incentives to do so are higher, thereby entrenching dominance. We explain the historical experiences of the Dutch florin, the British pound sterling, and the US dollar, and the transitions between them. Our theory highlights normative features of liquidity provision in the international monetary system through the lens of the Bretton Woods arrangement, and we discuss the implications of modern policy tools such as central bank swap lines. We rationalize the current dollar-dominant international financial architecture and provide predictions about the potential rise of the Chinese renminbi.

Keywords: International Monetary System, Dollar Dominance, Liquidity, Corporate Debt.
1 Introduction

The international monetary system has featured few but persistent dominant currencies in the denomination of financial contracts. The first dominant currency, the Dutch florin of the 17th century (Quinn and Roberds 2014b), was followed by the British pound sterling in the 19th century (Lindert 1969; Eichengreen 2005), which in turn was replaced by the US dollar in the 20th century. This paper posits a theory for why, among many alternatives for debt denomination, one currency emerges endogenously as dominant in equilibrium and is able to retain that dominance for prolonged periods. Our theory proposes that debt market liquidity is a key component of the factors establishing currency dominance in the denomination of global debt contracts.

We begin by observing that when borrowers such as firms issue debt, they must choose a unit of denomination for the contract. This choice determines the asset that the borrower must hand over to extinguish the debt at the time of settlement. The issuer, for example a farmer, could issue debt that is denominated in any arbitrary unit, such as bushels of wheat. In this case, upon maturity the farmer would be obliged to deliver bushels of wheat. But if bushels of wheat are difficult to come by—that is, if the wheat market is illiquid—this would be a costly decision. Perhaps a wheat farmer would issue such debt, but a manufacturer would not. Hence the settlement benefits of asset market liquidity, which we model via endogenous search frictions (Duffie, Gârleanu and Pedersen 2005), constitute the key economic force in our model.

In today’s international monetary system, the appeal of issuing debt denominated in dollars is the large and liquid nature of the dollar money market that facilitates dollar settlement. All borrowers worldwide can see that there is a significant number of investors who own a substantial amount of dollar-denominated, short-term money market instruments such as Treasury bills, repos, or high-grade bank and firm debt. As a result, issuers benefit from being able to easily trade their revenue streams with these investors to acquire the assets necessary to settle their debt obligations.

The ease of settling obligations in dollars incentivizes borrowers to choose dollars as the denomination of the debt contract at issuance. As more dollar debt is issued, some of this issuance is sufficiently safe and hence expands the supply of dollar-denominated money market assets that can be used by other borrowers to extinguish their own debt. Some

\footnote{Dollar dominance has many features including dollar-denominated debt outstanding in the world being outsized relative to the wealth or GDP share of the United States (see Cetorelli and Goldberg 2012; Bruno and Shin 2015b; McCauley, McGuire and Sushko 2015; Maggiori, Neiman and Schreger 2020).}
debt issuance, on the other hand, may not be safe enough to act as settlement instruments themselves, but they still expand the demand for future settlement and the returns to safe issuance. Together, dollar debt in these various forms begets more dollar debt, bootstrapping itself, leading to currency dominance.

The first dominant currency, the Dutch florin, is a useful example for highlighting the central way in which asset market liquidity benefits debt issuers. During this period, Spain was the largest and wealthiest economy with the highest volumes of trade, and the Spanish real de ocho (“pieces of eight”) was the most common coin in the world. However, the quantity and quality of these coins, like all other metallic coins, varied across time and space, limiting their settlement benefits. The Bank of Amsterdam harnessed a financial technology to create a new currency called the “bank florin” as a denomination that existed purely on the Bank’s ledgers (and hence was not subject to frictions associated with transport, insurance, and degradation), and which could be used to settle obligations via free account transfers. Parties around the world, even those not transacting directly with Amsterdam, typically chose florin denomination over the currency of the largest economy at the time, Spain, reflecting the value of the florin’s superior liquidity coming from its ledger-based settlement technology.\(^2\)

We formalize this liquidity force for dominance by modeling a financial demand for liquidity, similar to Holmström and Tirole (1998), whereby assets are needed to settle debt obligations. At the same time, asset market liquidity depends endogenously via search frictions on the quantity of available liquid bonds, as in Duffie, Gârleanu and Pedersen (2005). The key decision we study is that of a private borrower—a firm, for short—whose revenues are in a home currency that must choose the denomination of its debt in either the home or foreign currency. Choosing to denominate in foreign currency leads to costs associated with currency mismatch in the firm’s revenues and liabilities. Firms weigh this cost against the two benefits of issuing in a more liquid foreign currency: the ease of settling its own obligations and the convenience yield that accrues for issuing safe debt that can be used as a means of settlement by others, which provides more favorable borrowing terms.

First, by generating both benefits to issuance, our theory explains why empirically, there is a wide variety of borrowers that issue debt in the dominant currency in the cross-section. In today’s dollar world, some international firms have low credit ratings and hence

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\(^2\)The florin’s international dominance was evident in the fact that all foreign exchange rates were quoted in relation to it, and in certain places such as Russia, only the exchange rate relative to Amsterdam was quoted until 1763 (Van Dillen, 1934, p. 105). Moreover, the trade between England and Russia was conducted exclusively through payments in Amsterdam, reflecting how the “[Bank of Amsterdam was] the clearinghouse of world trade’ (De Vries and Van der Woude, 1997, p. 87, 131).
their bonds are not sufficiently money-like to be used as settlement liquidity by other parties, but these borrowers still benefit from the settlement liquidity in the dollar money market. Other borrowers, say a safe international issuer such as the German government-backed supranational bank KfW, likely do not have as many settlement needs, but nonetheless issue dollar debt to capture the convenience yield. Historically, we observe a wide range of firms and sovereigns issuing pound-denominated debt in London, some of which was safe and some other which was prone to default. In our model, the issuance by one type of borrower raises the benefits of issuance for the other such that liquidity supply and liquidity demand are complementary, endogenously reinforcing the dominance equilibrium.

Second, our model emphasizes that large asymmetries in financial market liquidity generate a unique dominant equilibrium. Historically, this asymmetry has come from large changes in the float of safe short-term government-backed liabilities. The government backing has taken various forms, whether through physical reserve holdings or via fiscal commitments. At the Bank of Amsterdam, confidence in the City of Amsterdam’s commitment not to appropriate the specie reserves that backed the florin was crucial for enabling it to implement its ledger technology, thereby creating a large float of florin. In contrast, Spain’s history of serial defaults made it unable to provide such a commitment. Similarly, the pound sterling’s dominance was established after Dutch losses in the Fourth Anglo-Dutch war caused a collapse in confidence in Amsterdam and the British won the Napoleonic Wars. At that point, British debt became the safest in the world, backed by its fiscal capacity. In the post-Bretton Woods period, the United States government has backed a large quantity of dollar-denominated short-term debt instruments in the form of Treasury bills, with more safe float than the government bonds of alternative currencies.

Our theory clarifies that the role of economic size in generating dominance is through its impact on financial market liquidity, but that size is not itself a fundamental determinant of dominance. Increasing the size of the private sector in the non-dominant country can in fact entrench dominance, rather than leveling the field, because it may be optimal for those firms to stay in the dominant equilibrium, thereby increasing supply and liquidity in the dominant currency. In our model, as was often the case historically, the dominant currency may not be hosted by the country with the largest GDP. For example, the florin was dominant despite Amsterdam being several times smaller than Spain. Likewise, in the late 19th century, the United States surpassed the United Kingdom in economic size, but American firms continued to issue bonds in London in sterling rather than domestically in dollars. In line with our focus on asymmetry in financial market depth, the shift from sterling to the dollar only
occurred after 1913, when the US increased its issuance of government debt and the creation of the Federal Reserve system pooled different regional financial centers into a central market that helped enhance the dollar money market’s liquidity.

Third, our model, with its dual issuance benefits, provides novel predictions for the impact of increasing the supply of safe government debt on the issuance incentives of other borrowers. We show that within a given dominant equilibrium, for a reasonable set of parameters, safe government debt issuance crowds out safe private debt issuance, as has been empirically documented in Krishnamurthy and Vissing-Jorgensen (2012), while crowding in risky debt issuance. This result contrasts with other work that does not distinguish between different types of private debt, and it allows us to explain the wide cross-section of firm issuance in the data, particularly that carried out by those borrowers that do not earn a convenience yield.

Fourth, our model shows that the country hosting the dominant currency has a greater capacity to invest in institutions that expand fiscal capacity and financial market liquidity, thereby generating an additional source of complementarity that entrenches dominance and leads to the persistence of regimes. Consider a costly action that expands a country’s fiscal backing and allows for the issuance of more safe assets. The convenience yield from debt issuance, which is seigniorage revenue for the government, is higher in the dominant currency. In addition, the incentive to undertake this action is lower for the non-dominant country since seigniorage revenues are lower and additionally, liquidity expansion in the non-dominant currency does not benefit firms that have already chosen the dominant currency. It is therefore beneficial for the dominant country to invest more in costly actions that create more fiscal backing—for example, by investing in the country’s military to capture more resources. This expansion allows for more debt issuance, which endogenously attracts more firms to the dominant currency. Dominance facilitates investments that further entrench the dominant regime, leading to persistence. Historically, dominant country investments for example in Amsterdam and the UK have been directed towards financial and legal institutions as well as military capacity.

Fifth, international trade activity also allows the possibility that firms receive their revenues in the foreign currency (through trade invoicing). This reduction in the costs of dominant currency debt denomination reinforces dominance in a complementarity first modeled by Gopinath and Stein (2021) and Chahrour and Valchev (2022). As in these models, we also rationalize the joint observation of trade and finance in a dominant currency. Our model, however, highlights why the volume of trade need not be the primary driver of
the dominant currency paradigm, as we have seen in previous historical episodes.

Sixth, our theory sheds light on normative aspects of liquidity provision arrangements in the international monetary system. The decision to denominate debt in the dominant currency carries a positive externality, as it improves market thickness for all other borrowers and lenders. As a result, the competitive equilibrium is not efficient: a global planner will want to subsidize even more entry in the dominant currency. Socially beneficial coordination can be implemented with formal international arrangements, like the Bretton Woods system, in which the commitment devices backing a country’s monetary liabilities (such as gold) are concentrated in a single country, while all other countries have no formal need for gold.

Finally, we extend the model to include default risk and liquidity provision policy tools. At least since the Bretton Woods arrangement, the US has committed to state-contingent expansions of dollar liquidity via central bank swap lines. The Federal Reserve provided these swap lines in several recent crises when global banks and foreign central banks exhibited large demands for dollars to settle their firms’ liabilities. Our model shows that such expansions help enhance dominance. We introduce aggregate shocks to firms’ liquidity demand and allow the government to tailor liquidity supply to depend on the aggregate state. State-contingent policies that generate positive covariance between liquidity supply and liquidity demand, such as central bank swap lines, make the dominant currency more attractive. By the same token, however, if liquidity supply falls in a high liquidity demand state—negative covariance—this can undercut a currency’s appeal. This latter point helps explain why the Euro has lost reserve currency ground to the dollar after the 2008 financial crisis (Maggiori et al., 2020), when Euro-area sovereign debt downgrades were followed by a sharp contraction in the international use of the euro.

**Related Literature.** Our paper is most directly related to work that explains the emergence of a dominant unit of account in financial contracts.³ Doepke and Schneider (2017) presents a model where agents in a chain of production choose the denomination of contracts for goods purchases and sales. In this setting, it is efficient to avoid asset-liability mismatch that could lead to costly default by coordinating on a single denomination of these contracts. Furthermore, if there is a sufficient quantity of government debt that agents hold as assets, then the unit of denomination of this debt will be chosen as the unit in private contracts. In the international setting, Gopinath and Stein (2021) and Chahrour and Valchev (2022) con-

³There is a related but separate literature on the medium of exchange role of money (Kiyotaki and Wright, 1989), and in the international context, the rise of a vehicle currency as a medium of exchange (Matsuyama et al., 1993).
sider a model where agents simultaneously choose the unit of denomination of their revenues (trade invoicing) and of their debts (liability invoicing). Given the need to finance imported goods, coordinating assets and liabilities in the same unit of denomination is optimal.

The cost side in the debt denomination decision, i.e. currency mismatch, is the same in our paper as in those cited above. However, our paper highlights a new channel for the benefits of denoting financial contracts in the dominant currency, which stems from ease of settlement in financial assets. In practice, the volume of financial flows is far larger than the volume of flows in goods. Thus our model generates the complementarity leading to currency dominance via purely financial factors. There is an empirical literature pointing to the real effects of denoting financial liabilities in foreign currencies (Aguiar, 2005; Verner and Gyöngyösi, 2020; Adams and Verdelhan, 2022). This literature indicates that a wide variety of firms, including non-exporters, choose foreign currency debt denomination and are rarely fully hedged against exchange rate fluctuations. We provide a model of the liquidity benefits of choosing this foreign currency exposure in equilibrium, which rationalizes this denomination pattern.

Maggiori (2017) and Gourinchas and Rey (2022) present models where the rest-of-the-world is effectively more risk-averse than the US, generating a world demand for safe (dollar) bonds and a low interest rate on US (dollar) bonds. In equilibrium, the center country (the US) issues (dollar) bonds to the rest-of-the-world. While our model also explains why firms in the center country issue debt in the dominant currency, it additionally explains why entities outside the center country have also historically been large issuers of debt in the reserve currency. In Gopinath and Stein (2021), the demand for dollar-denominated consumption drives demand for safe dollar claims, which creates a convenience yield on safe dollar debt and thus explains why safe firms and banks around the world issue dollar debt. Our model explains why risky firms whose debt may not carry a convenience yield, but who still benefit from foreign currency liquidity at settlement, also choose dominant currency denomination.

The model also sheds light on the history of reserve currencies, incentives of the sovereign, and the international monetary system. Work on the nature of the international monetary system, which we relate to, includes Nurske (1944), Obstfeld et al. (1995), Tirole (2002), Gourinchas and Rey (2007), Farhi and Maggiori (2018), and He et al. (2019). Quinn

Hassan (2013) presents a model where the large size of the US endogenously leads to a low interest rate on dollar bonds. Eren and Malamud (2022) present empirical evidence showing that over longer horizons the dollar depreciates after negative shocks, and this can make it optimal for firms to issue debt in dollars as a hedge against the downturns. 
and Roberds (2014b,a) and Bolt et al. (2023) describe the experience of the Dutch florin in the 17th and 18th centuries. Similarly, King (1972), Dickson (1967), Lindert (1969), Eichengreen (2008), and Kynaston (2015a,b) examine British pound dominance in the early 19th and early 20th centuries. Krugman (1984), Frankel (1992), Bruno and Shin (2015a,b), Ivashina et al. (2015), Bahaj and Reis (2020), Maggiore et al. (2020), Correa et al. (2022), and Jiang et al. (2022) examine different aspects of the current dollar-dominant regime in international finance. Our theory supports the narrative of Eichengreen (2012) and Eichengreen et al. (2017) that throughout many historical episodes, financial development in the center country that deepened financial markets played crucial roles in supporting currency dominance. In addition, we highlight how financial market depth is complementary to investments in fiscal capacity.

Our paper also belongs to the literature on the role of safe assets in the economy. Theoretical work in this area explores the macroeconomic and asset pricing implications of safe asset shortages (Holmström and Tirole 1998; Caballero et al. 2008; Caballero and Krishnamurthy 2009), and fiscal limitations in the creation of safe assets (Farhi et al., 2011; Obstfeld, 2012). There is also an empirical literature documenting how the supply of safe impacts asset prices and quantities (Krishnamurthy and Vissing-Jorgensen 2012; Gorton et al. 2012; Greenwood et al. 2015). Our model shows that the supply and denomination of safe assets affects the determination of the dominant currency and the convenience yields of safe assets denominated in that currency. Dollar safe assets carry a convenience yield (Du et al., 2018; Jiang et al., 2021), and dollar safe assets are held as reserve assets by central banks around the world (Ilzetzki et al., 2019). Our model predicts that the growing accumulation of dollar reserves by central banks around the world reflects these institutions holding the assets necessary to facilitate liquidity provision in the denomination of firms’ financial obligations.\footnote{One can also interpret our model to shed light on the dominance of exchange rate pegs to the dollar and foreign central bank accumulation of dollar reserves (Ilzetzki et al., 2019) via this logic. By pegging the exchange rate, the central bank reduces currency mismatch costs to local borrowers, while enabling them to benefit from issuing in the liquid dollar market. But to maintain the peg, the central bank accumulates dollar assets and trades these dollars to local firms for their goods’ revenues when needed. The local firms then use the dollars to settle their dollar liabilities. The demand for these reserve assets coming from settlement needs generates a convenience yield on both public and private safe dollar claims.}

We model liquidity using the search framework of Duffie et al. (2005). Our theory features increasing returns to scale in search, which, as emphasized by Weill (2020), is a well-supported characterization of financial markets. Other literature featuring increasing returns to scale in financial markets includes Pagano (1989), Duffie et al. (2007), Garleanu and Pedersen (2007), Vayanos and Wang (2007), Vayanos and Weill (2008), Weill (2008),
Lagos and Rocheteau (2009), Shen et al. (2018), Sambalaibat (2022), Geromichalos and Herrenbrueck (2022), and Geromichalos et al. (2023). Further, we connect to work highlighting the importance of search frictions in sovereign debt pricing and exchange rate determination, including Chaumont (2020), Moretti (2020), Bianchi et al. (2021), and Passadore and Xu (2022). We trace out the implications of these search-based illiquidity frictions for the corporate financing decisions of firms around the world.

2 A Model of Liquidity and Debt Denomination

We consider a three-period ($t = t_0, t_1, t_2$) environment with two countries, indexed by $j \in \{A, B\}$. In each country, there is government-backed risk-free debt of mass $G_j$ denominated in units of the local currency, which can be traded to meet the liquidity needs of agents. The mass $G_j$ includes all money-like instruments denominated in currency $j$ that the government has committed to guarantee.

There are also entrepreneurs who run firms that issue safe debt (e.g., bills of exchange or corporate bonds), in mass $F_j$. These private borrowers make a choice of denominated the debt in their home currency or in the foreign currency. Firms have liquidity needs in a manner similar to Holmström and Tirole (1998). Asset trading occurs in a secondary market with endogenous trading frictions, as in Duffie et al. (2005). Finally, there is a continuum of homogeneous risk-neutral investors that buy the debt of firms and governments.

2.1 Within-Country Environment

We start by developing the model within a given country, where we consider the choice to issue debt of private-sector borrowers. In Section 2.2, we turn to the full case where firms may choose the currency in which to issue debt and characterize the general equilibrium.

2.1.1 Debt Issuance at $t_0$

There is a mass of entrepreneurs $F_j$ in each country $j$. Each entrepreneur owns a firm that can borrow to invest in a project at $t_0$. The project will generate profits of one at a stochastic time, either $t_1$ or $t_2$. The investment has a cost $\beta^2$, which is incurred at $t_0$. At $t_0$, the entrepreneur can raise funds for the investment by selling debt with face value of one maturing at $t_2$, which will be repaid using the future profits. As will become clear, the model is set up so that borrowing and investment is always profitable for the entrepreneur.
The preference of a given entrepreneur $i$ is to maximize:

$$u_{i,j}^F = c_0 + \beta c_1 + \beta^2 c_2, \quad c_t \geq 0, \quad \beta < 1. \quad (1)$$

At $t_0$, the government also issues a quantity $G_j$ of government-backed securities. We assume that both government and private bonds are safe (i.e., there is no default risk, as we elaborate on in Section 2.3), and that they have the same liquidity properties—hence, they are priced the same in equilibrium. We consider breaking this symmetry later in the analysis. We also consider the case of risky bonds later in the analysis. For now, as the bonds are identical they have the same endogenous price $P_{0,j}$.

There is a mass $I_j$ of investors who have sufficiently large endowments to purchase bonds issued by the government and firms at $t_0$. The investors are risk neutral with preferences

$$u_{i,j}^I = c_0 + \beta c_1 + \beta^2 c_2, \quad c_t \geq 0. \quad (2)$$

Each investor potentially owns one bond, and bonds are indivisible. The total mass of bonds is $G_j + F_j$. Define the total mass of bondholders to be

$$m_{I,j} = G_j + F_j \leq I_j, \quad (3)$$

where the last inequality is a restriction on parameters. That is, we assume there are enough investors to purchase all of the bonds at $t_0$.

### 2.1.2 Money Market Settlement With Search Frictions

The liquidity need in the model arises if the entrepreneur’s profits arrive early at $t_1$ while his debt is due at $t_2$. In this case, his debt and profits streams are mismatched in time. The probability of receiving profits early is $\phi$. The mass of liquidity-demanding firms is the total number of early firms, which, by the law of large numbers, is

$$m_{F,j} = \phi F_j. \quad (4)$$

With the possibility of early revenues, it may be beneficial for firms to obtain a financial asset at $t_1$ because such an asset would allow them to have a savings vehicle to extinguish their $t_2$ debt obligations. The financial assets that firms seek are the bonds issued by the government and other firms at date $t_0$. We model obtaining financial assets from the money market as search and matching.
Firms with early profits may trade with investors at \( t_1 \) to obtain the financial asset (i.e., a bond) for settlement at \( t_2 \). There are gains from trade in a meeting. We assume that if the firm does not trade with the investor, then it keeps its profits until \( t_2 \) and uses the profits to settle its debt. The effective return on keeping its profits is therefore zero. However, since investors discount the future at rate \( \beta < 1 \), an investor who owns a bond is willing to sell the bond as long as he receives at least a quantity \( \beta \) of goods that he consumes at \( t_1 \). The gains from trade in a match between investor and firm is therefore \( 1 - \beta \). We assume the firm receives a fraction \( \eta \) of this surplus and the investor keeps the remaining \( 1 - \eta \) share.

We now describe the search market at \( t_1 \). We posit a matching function such that the number of meetings between liquidity demanders (firms) and liquidity suppliers (date \( t_0 \) investors) is

\[
n_j = \lambda_j m_{F,j}^\theta m_{I,j}^\theta, \quad \theta > \frac{1}{2}.
\]

Here \( \lambda_j > 0 \) captures the overall degree of liquidity of the money market. In the continuous-time asset trading model of Duffie et al. (2005, henceforth DGP), \( \lambda_j \) corresponds to the Poisson probability that a given agent (say, a firm) will meet another agent (say, an investor). Given Poisson meeting rates, \( \theta = 1 \) so that the total number of matches is proportional to the masses of both firms and investors (Duffie et al., 2018).

The key property of this matching function is increasing returns to scale, corresponding to \( \theta > \frac{1}{2} \). If the masses of both firms and bond-holding investors double, the number of matches more than doubles. Thus the search model embeds a thick-market liquidity externality as in Diamond (1982). This liquidity externality is at the heart of many of our results. We focus our analytical derivation on the case of \( \theta = 1 \). We also provide numerical results for the general \( \theta \in (1/2, 1) \) case.

Given the matching function, the endogenous two-sided meeting probabilities are:

\[
\alpha_{F,j} = \frac{n_j}{m_{F,j}} = \lambda_j m_{F,j}^{\theta-1} m_{I,j}^\theta, \quad \alpha_{I,j} = \frac{n_j}{m_{I,j}} = \lambda_j m_{F,j}^\theta m_{I,j}^{\theta-1}.
\]

Here \( \alpha_{F,j} \) is the probability of a firm meeting a bond seller (date \( t_0 \) investor in bonds) at time \( t_1 \), and \( \alpha_{I,j} \) is the probability that the bond seller meets a firm. The key economic force that the modeling captures is that the trade to obtain assets for settlement is frictional, and that a greater outstanding quantity of bonds makes obtaining this liquidity easier (higher \( \alpha_{F,j} \)). Figure 1 provides a timeline of the debt market.

The date \( t_1 \) liquidity market as described is an over-the-counter (OTC) bond market.
Figure 1: Timeline of debt issuance and demand for settlement

Notes: This figure presents a schematic representation of timing in the model.

as in Duffie et al. (2005), where firms trade goods with investors for their one-period bonds. However, we do not take a stand on the market structure of this trade, and it is likely that this structure has varied across history, as we describe in Section 2.3.

2.1.3 Firm Issuance Decision and Asset Market Equilibrium

We formally present the entrepreneur’s decision problem. The entrepreneur makes an issuance decision at date $t_0$. Denote $D_i$ as an indicator function that takes the value one if the firm issues debt to invest and zero if the firm does not. The firm decides at date $t_1$ to trade for a bond or not. Denote $T_i$ as an indicator function that reflects the decision to trade. Then the entrepreneur’s problem is:

$$\max_{D_i, T_i} E[c_0 + \beta c_1 + \beta^2 c_2]. \quad (7)$$

The structure we have posited imposes several constraints on this problem. Consumption at date $t_0$ is $c_0 = D_i (P_{0,j} - \beta^2)$. Consumption at date $t_1$ is $c_1 = D_i T_i \eta (1 - \beta)$ if the profits arrive early and the firm matches with a counterparty in the asset market, and zero otherwise. Consumption at date $t_2$ is always zero in equilibrium, $c_2 = 0$. Then, as long as $P_{0,j} \geq \beta^2$ and $1 - \beta > 0$, the solution is to set $D_i = 1$ and $T_i = 1$.

We solve for $P_{0,j}$ backwards. Consider the market at date $t_1$ first. If a match occurs, the total surplus is $1 - \beta$, of which a bond seller obtains $(1 - \eta)(1 - \beta)$. We assume that the date $t_0$ bond market is Walrasian. Each investor can bid for exactly one bond at date $t_0$. If an investor purchases a bond at $t_0$, the investor either resells the bond at date $t_1$ to earn $\beta + (1 - \eta)(1 - \beta)$, or the investor holds the bond to maturity. Thus the investor’s valuation of the bond at $t_0$ is:

$$P_{0,j} = \alpha_{I,j} \beta [\beta + (1 - \eta)(1 - \beta)] + (1 - \alpha_{I,j}) \beta^2 \frac{P_{\text{Matched}} \times \text{PV of Sales Price} + (1 - \alpha_{I,j}) \beta^2}{P_{\text{Not Matched}} \times \text{PV of 1}}, \quad (8)$$
or rewriting,
\[ P_{0,j} = \beta^2 + \alpha_{I,j} \beta (1-\eta)(1-\beta). \]  
(9)

Since \( 1 - \eta > 0 \), we have that \( P_{0,j} > \beta^2 \), so that \( D_i = 1 \) in the firm’s issuance problem.

We define the wedge \( P_{0,j} - \beta^2 \) as a *convenience yield* on bonds issued at \( t_0 \). Consider the pricing of a completely illiquid bond, which in our model is one for which \( \alpha_{I,j} = 0 \). This bond will be priced at \( \beta^2 \). The government and private firm bonds in our model are priced at \( P_{0,j} > \beta^2 \) because they offer settlement liquidity to firms at date \( t_1 \). The convenience yield increases in the match probability \( (\alpha_{I,j}) \) and the surplus gained from the match \(( (1-\eta)(1-\beta) ) \).

Finally, consider the entrepreneur’s expected utility from bond issuance at date \( t_0 \) at an endogenous price \( P_{0,j} \):

\[
u_{F,i,j} = P_{0,j} - \beta^2 + \beta \phi \alpha_{F,j} \times \eta (1-\beta).
\] (10)

The first two terms in this objective reflects the benefit from selling bonds at a high price at \( t_0 \) minus the cost of investment, which is the convenience yield. The second term reflects the benefit of settlement liquidity at \( t_1 \). The firm is early with probability \( \phi \) and obtains the needed liquidity with probability \( \alpha_{F,j} \). The share of the surplus in the trade that the firm receives is \( \eta (1-\beta) \), and the firm discounts the future at \( \beta \).

Given equilibrium bond prices, we can rewrite (10) as:

\[
u_{F,i,j} = \lambda_j \beta (1-\beta) (m_{F,j} m_{I,j})^{-1} [(1-\eta)m_{F,j} + \phi \eta m_{I,j}] .
\] (11)

The two additive terms in this expression reflect the two ways in which firms benefit from money market liquidity: the first term reflects the benefit of capturing convenience yields on the firms’ initial issuance, which is increasing in \( m_{F,j} \), the mass of firms demanding settlement liquidity, and in \( 1-\eta \), the surplus share accruing to the owner of the bond if a trade happens. The second term reflects the benefit from a high probability of being able to find a match in the date \( t_1 \) money market, which is increasing in the mass of available bonds \( m_{I,j} \), in the surplus share \( \eta \) going to the firm needing settlement, and in \( \phi \), the probability that the firm needs settlement.

In equation (11), both benefits accrue to the same issuers because firms are homogeneous. In practice, certain issuers can be best thought of as pure liquidity providers who harvest the convenience yield but do not have large transaction needs. On the other hand,
different issuers may not be able to capture convenience yields because their debt does not offer sufficient liquidity benefits (e.g., because of default risk), but may still value settlement liquidity—and hence may be best thought of as pure liquidity demanders. In Section 3.4, we consider an extension in which we separate these two roles in the cross-section of firms.

From now on, we set $\eta = \frac{1}{2}$, as we do not explicitly model the bargaining process and $\eta$ plays no part in the analysis. A firm’s expected utility from debt issuance is therefore:

$$u^F_{i,j} = \frac{1}{2}\lambda_j \beta (1 - \beta) (m_{F,j}m_{I,j})^{\theta - 1} [m_{F,j} + \phi m_{I,j}].$$  \hspace{1cm} (12)

### 2.2 International Equilibrium Conditions

We next describe the international equilibrium. The two countries, $j = A,B$, have fundamentals $\{G_j, \lambda_j, F_j\}$. Firms earn revenues in domestic currency and choose the denomination of bonds, either domestic or foreign. We assume that the government only issues bonds in its domestic currency.

We assume that there is a fixed cost of issuing in a foreign currency proportional to $K_i$, which is heterogeneous across firms. In Appendix Section A.1, we provide a microfoundation in terms of costs associated with running a balance sheet currency mismatch. We assume that $K_i$ is distributed on $[K, \infty)$ with cumulative distribution function $H(K_i)$, and corresponding density $h(K_i) = H'(K_i)$. This density is identical in the two countries.

We let the set of buyer and seller masses be $M = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$. We can compute expected utility for the entrepreneurs in the two countries and for each of the possible denomination choices. These four expressions are as follows:

1. **Expected utility of entrepreneur in country B issuing in foreign currency (A):**

$$U_{B\rightarrow A}(M, K_i) \equiv \frac{\beta(1 - \beta)}{2} \lambda_A (m_{F,A}m_{I,A})^{\theta - 1} [m_{F,A} + \phi m_{I,A}] - K_i.$$  \hspace{1cm} (13)

2. **Expected utility of entrepreneur in country B issuing in home currency (B):**

$$U_{B\rightarrow B}(M) \equiv \frac{\beta(1 - \beta)}{2} \lambda_B (m_{F,B}m_{I,B})^{\theta - 1} [m_{F,B} + \phi m_{I,B}].$$  \hspace{1cm} (14)

One can also alternatively think of the cost $K_i$ as capturing the costs of hedging foreign currency exposures. In addition, $K_i$ might also incorporate fixed costs of finding a bank and underwriters of bonds in the foreign currency, providing information so that foreign investors can assess the firms’ risks, and so on.
3. Expected utility of entrepreneur in country A issuing in foreign currency (B):

\[ U_{A\rightarrow B}(M, K_i) \equiv \frac{\beta(1-\beta)}{2} \left[ \lambda_B (m_{F,B}m_{I,B})^{\theta-1} [m_{F,B} + \phi m_{I,B}] - K_i \right]. \] (15)

4. Expected utility of entrepreneur in country A issuing in home currency (A):

\[ U_{A\rightarrow A}(M) \equiv \frac{\beta(1-\beta)}{2} \lambda_A (m_{F,A}m_{I,A})^{\theta-1} [m_{F,A} + \phi m_{I,A}]. \] (16)

We index denomination choice by \( D_{i,j} \), where \( D_{i,j} = 1 \) if firm \( i \) in country \( j \) issues in foreign currency, and \( D_{i,j} = 0 \) otherwise. Each firm chooses its debt denomination optimally by comparing the expected utility functions given above:\(^7\)

\[ D_{i,j} = \begin{cases} 
1 & \text{if } U_{j\rightarrow j'}(M, K_i) > U_{j\rightarrow j}(M), \\
0 & \text{if } U_{j\rightarrow j'}(M, K_i) \leq U_{j\rightarrow j}(M). 
\end{cases} \] (17)

We then have three results. First, since \( U_{j\rightarrow j'} \) is monotonically decreasing in \( K_i \), we obtain:

**Lemma 1.** Consider firms \( \hat{i} \) and \( i \) in country \( j \), where \( K_i < K_{\hat{i}} \). If it is optimal for firm \( \hat{i} \) to issue in foreign currency \( j' \neq j \), then it is also optimal for firm \( i \) to issue in foreign currency \( j' \).

Next, notice that the expressions for firm utility in country A have the same terms as the expressions for firms in country B. As a result:

**Lemma 2.** Suppose that there is a positive mass of firms in \( j \) that find it optimal to issue in currency \( j' \). Then, no firms in \( j' \) will issue in currency \( j \).

In other words, if some firms in B choose to pay a cost to issue in country A, then a firm in A for which there is no cost to issue in A will choose to only issue in A, and hence no firm from A issues in B. Together, these two lemmas imply that optimal firm denomination choices must have a threshold structure, which we formalize as follows.

---

\(^7\)In writing a B firm’s problem as one of comparing the expressions for utility to issuing in A to that of B, we require that conditional on a currency choice the optimal choices over issuance and settlement are as described in the previous within-country setting. With two currencies, there is another action that is feasible: a B firm could issue in currency B, then at date \( t_1 \) the early-firm converts its goods into currency A and searches for a bond in A, and finally at \( t_2 \), the firm converts its bond proceeds back into B and uses these proceeds for settlement. We assume that these two further foreign exchange transactions expose the firm to currency risk and thus the firm incurs additional costs. In this case, while this action is feasible, it is never optimal.
Lemma 3. A necessary condition for a collection of firm denominations choices $D_{i,j}$ to be consistent with firm optimality is that it must take the following threshold form:

$$D_{i,j'} = \begin{cases} 
1 & \text{if } K_i < \bar{K}, \\
0 & \text{if } K_i \geq \bar{K}, 
\end{cases} \quad D_{i,j} = 0. \quad (18)$$

In the lemma above, $\bar{K}$ corresponds to the threshold cost below which firms choose to issue in foreign currency: it is a scalar that provides a sufficient statistic summarizing the entirety of the set of all firms’ denomination choices. A corollary is that the masses $M$ can themselves be represented as functions of the threshold cost $\bar{K}$: $M = M(\bar{K})$. Further, we introduce the following notation for the expected utilities of the threshold firm (for which $K_i = \bar{K}$):

$$\bar{U}_{j'\to j}(\bar{K}) \equiv U_{j'\to j}(M(\bar{K}), \bar{K}), \quad \bar{U}_{j'\to j'}(\bar{K}) \equiv U_{j'\to j'}(M(\bar{K})). \quad (19)$$

We denote the equilibrium value of $\bar{K}$ as $\hat{K}$. This is the cost which sets the marginal firm indifferent between the two currencies, equalizing the two expressions in equation (19).

Given the threshold structure for firms strategies, we can formally define equilibria.

Definition 1. An equilibrium is a collection of an endogenous threshold $\hat{K}$, exogenous parameters $\Theta = (F_A, F_B, G_A, G_B, \lambda_A, \lambda_B, \phi, \theta)$, an exogenous firm size distribution $H(K)$, and endogenous masses $M = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$, satisfying:

1. (Market clearing) Given $\hat{K}$, the masses $M$ satisfy:

$$m_{F,j} = \phi \left[ F_j + H(\hat{K}) F_{j'} \right], \quad m_{F,j'} = \phi \left( 1 - H(\hat{K}) \right) F_{j'}, \quad (20)$$

and,

$$m_{I,j} = G_j + F_j + H(\hat{K}) F_{j'}, \quad m_{I,j'} = G_{j'} + \left( 1 - H(\hat{K}) \right) F_{j'}. \quad (21)$$

2. (Firm optimality) Given the masses $M$, the threshold is optimal: firms in $j'$ with $K_i < \hat{K}$ find it optimal to issue in currency $j$, while all other firms optimally issue in their own currency, so that

$$\begin{cases} 
\bar{U}_{j'\to j}(\hat{K}) = \bar{U}_{j'\to j'}(\hat{K}) & \text{for } \hat{K} > K, \\
\bar{U}_{j'\to j}(\hat{K}) \leq \bar{U}_{j'\to j'}(\hat{K}) & \text{for } \hat{K} = K. 
\end{cases} \quad (22)$$

\[^8\text{Alternatively, by complementary slackness, equilibria at the lower boundary (} \hat{K} = K \text{) are also valid if } \bar{U}_{j'\to j}(K) \leq \bar{U}_{j'\to j'}(K).\]
Further, throughout the analysis, we will examine the stability properties of the equilibria specified in the above definition, whenever the model features multiple equilibria. We formally specify our notion of stability using the following criterion.

**Definition 2.** Consider an underlying dynamical system through which an out-of-equilibrium system converges to equilibrium (a tatonnement process) of the form $\partial_\tau \hat{K} = \delta \left[ U_j(\hat{K}) - U_j'(\hat{K}) \right]$ for $\delta > 0$ and where $\tau$ indexes a mass of continuous sub-periods within time $t_0$. An equilibrium featuring the endogenous threshold $\hat{K}$ is said to be stable if it fulfills the following condition:

1. (Equilibrium stability) There exists an $\varepsilon > 0$ such that any trajectory beginning in the neighborhood $[\hat{K} - \varepsilon, \hat{K} + \varepsilon]$ converges to $\hat{K}$.

### 2.3 Discussion of Modeling Choices

We further explain some of the modeling choices that we have made in this section.

1. *Rollover risk vs. saving:* The liquidity trade in our model at $t_1$ is firm “saving”, but this is more for simplicity than it is substantive. The key component is that the firm trades its goods for a one-period bond. Consider a variant of the model in which there is a date $t_3$, and the firm’s liquidity need stems from rollover risk. The timing mismatch in that problem is that with some probability, the firm will receive the goods at $t_3$ rather than $t_2$ while the debt is due at $t_2$. The firm will then want to trade its future revenues for bonds at $t_1$. If there are more bonds available, this trade will be less frictional and it will be less costly to rollover the debt. Again, the key economics our model captures is in linking liquidity with bond supply.

2. *Interpretation of $G$.* While we define $G_j$ as the supply of safe government bonds, it should be more broadly interpreted as the supply of money market instruments that depends on the government’s commitment to maintain their value. Throughout history, government commitment has taken various forms: either purely fiscal, or through physical holdings of precious metals such as in a gold standard. For example, in the context of Amsterdam, the relevant government-backed debt supply is not the sovereign debt, but rather the specie-backed bank florin. The relevant government commitment for firms was the City of Amsterdam’s promise not to appropriate the underlying specie. When confidence in this commitment collapsed, as it did in 1795, there was a drop in $G$ even though ultimately the default rate on florin was low. The
relevance of the government commitment is also why Spain, despite having a large nominal debt in the 18th century, had low $G$. Its history of default reduced its ability to credibly commit to supporting a float of safe and liquid assets. In the modern context commitment generally takes the form of fiscal backing. We model government default risk in Section 4.5.

3. Institutional structure of the money market at $t_1$: The asset trade between investors and firms, in which investors that have bought safe debt at $t_0$ provide liquidity at $t_1$, often takes the form of financial intermediation. For example, in Amsterdam, a $t_0$ investment took the form of traders depositing their specie at the Bank to receive a florin deposit (Quinn and Roberds, 2014a). These traders would pay a fee to the Bank in this transaction and would therefore receive less florin than the value of the specie, as in an overcollateralized repo transaction. The traders would then lend florin to a merchant at $t_1$ who needed florin for settlement, earning interest or a fee in this transaction. From the standpoint of the model, the repo technology and specie are “liquidity supply” and the merchant is the “liquidity demander” at $t_1$.

These labels also apply in thinking about a banking arrangement or modern money market and repo. The “liquidity suppliers” ($m_I$ in the model) are banks that own the bonds as an asset and issue deposits to “liquidity demanding” firms ($m_F$) at $t_1$, which the firms then use to settle their $t_2$ debt. Alternatively, financial firms use Treasury securities as repo to obtain reserves that can then be lent to others as settlement instruments.

In the global dollar market, liquidity supply is provided by foreign central banks and global banks. These entities hold safe dollars as foreign reserves, obtained at $t_0$ in the form of Treasury securities, which they provide to firms at $t_1$. In this way, reserve holdings would tilt towards the dollar even without asymmetry in investor preferences for dollar assets—as in, for example Gourinchas and Rey (2022) or Jiang et al. (2020a).

4. Theoretical understanding of denomination: payoff risk vs. settlement. Currency denomination encompasses two distinct aspects: payoff risk and settlement. For example, a contract where payoffs vary with nominal exchange rates is one where denomination determines payoff risk. Our theory instead highlights the settlement aspect of denomination. To extinguish a dollar-denominated debt contract, the borrower needs to deliver dollars. As a concrete example, consider the payoffs of Swiss Franc or Canadian dollar debt. These currencies have exchange rates with similar payoff characteristics
as the dollar: they appreciate during periods of economic turmoil. Yet, in terms of quantities, dollar debt is many orders of magnitude larger than that of Canadian or Swiss debt. Our settlement-liquidity theory can speak to this fact, whereas a payoff risk theory likely cannot. Historically, the classical gold standard era provides similar examples. Currencies like the German mark and French franc also provided a claim to the same underlying specie payoff as the British pound sterling, and yet there was much less foreign debt issuance in marks or francs relative to pounds.

5. *Is search and liquidity a concern in money markets?* Our model links corporate financing decisions to money market illiquidity concerns. Our analysis turns on the relative liquidity of the money markets in different currencies rather than the absolute liquidity of any one market. At a theoretical level, our model builds on a long tradition of using search to model money markets (see Kiyotaki and Wright, 1993; Lagos and Wright, 2005). Empirically, even in high-volume money markets such as the US dollar repo market, search models have been shown to capture price and quantity patterns well (Vayanos and Weill, 2008). At the macro level, during a period of global financial volatility, the dollar money market remains more liquid than the markets of many emerging and even advanced economies. In our model, these considerations drive financing decisions.

### 3 Currency Dominance and Denomination Incentives

Having specified the model environment and derived its equilibrium conditions in Section 2, we now turn to analyzing the properties of the resulting equilibria and examining the underlying economic forces. We outline how multiple equilibria naturally emerge as a consequence of increasing returns to scale, and how asymmetries in country fundamentals favor those equilibria that feature currency dominance. We illustrate the complementary benefits to debt issuance in the cross-section of firms, and we provide novel predictions for the impact of government debt issuance on firm issuance incentives.

#### 3.1 Equilibria in the Symmetric $K_i = 0$ Case

Let us start by analyzing the simplest, fully symmetric case where fundamentals $(\lambda_j, G_j, F_j)$ are the same for both $j = A, B$, and where we also set the foreign-currency issuance cost $K_i$
to zero for all firms $i$.\footnote{Therefore, in this case the distribution of costs $K_i$ is degenerate with support only at zero: $H(K) = 1$ for $K \geq 0$, and $H(K) = 0$ otherwise.} We present analytical results for the $\theta = 1$ case.

**Proposition 1.** There are three equilibria in the symmetric, $K_i = 0$, $\theta = 1$ case:

1. No firm switches to issue in the foreign currency.
2. All firms from $B$ switch to issuing in currency $A$.
3. All firms from $A$ switch to issuing in currency $B$.

We refer to equilibria in which firms in country $B$ switch to currency $A$ (equilibrium 2 above) as “class $BA$ equilibria”, and to equilibria in which firms in country $A$ switch to currency $B$ (equilibrium 3 above) as “class $AB$ equilibria”.

Consider each of these cases in turn:

1. No firm switches, so that the masses are symmetric across $A$ and $B$. As a result the utility from issuing in the home currency equals that of issuing in the foreign currency. The convenience yields on bonds in both $A$ and $B$ are equal:

$$P_{0,j} - \beta^2 = \frac{\lambda_j \beta (1 - \beta)}{2} m_{F,j}.$$  \hfill (23)

2. All firms switch to $A$. Starting from the no-firms-switch case, if a small mass of firms were to shift from $B$ to $A$, then the masses $m_{F,A}$ and $m_{I,A}$ would rise while the corresponding masses in $B$ would fall. Then because of increasing returns to scale in matching, the liquidity benefit from issuing in $A$ rises relative to that of $B$. We can see this by examining the liquidity benefit of switching to $A$:

$$U_{B \to A} = \frac{\lambda_A \beta (1 - \beta)}{2} [m_{F,A} + \phi m_{I,A}].$$  \hfill (24)

Equation (24) highlights the two forces that drive firm decisions. Consider the second term in brackets. A firm that needs liquidity at date $t_1$, which occurs with probability $\phi$, benefits from having a larger pool of liquidity, which is linear in $m_{I,A}$. Next consider the first term in the bracket. At date $t_0$, firms sell their bonds at a convenience yield because these bonds are used for settlement purposes at date $t_1$. This convenience yield benefit is increasing in $m_{F,A}$, the mass of firms needing early settlement.
As the liquidity benefit of moving to $A$ rises, the liquidity benefit of remaining in $B$ falls. The result is that the equilibrium resolves with all firms switching to $A$. In this equilibrium, the convenience yield on bonds in $B$ is zero, while it is positive in $A$.

3. All firms switch to $B$. This is the symmetric case as that of (2).

The fully symmetric case illustrates the economics at work in the model. The dominant equilibria of all firms switching to either $A$ or $B$ are the only stable ones. The equilibrium in (1) is unstable: a movement in masses to either $A$ or $B$, such as from a perturbation in $G$, shifts the benefits for all other firms in the same direction leading to the all-firms-switch equilibria.

### 3.2 Incentives and Equilibria With Heterogeneous Costs

We next focus on a case where costs differ across firms. We parameterize $H(K)$ as a Pareto distribution, so that the cumulative distribution function takes the form $H(K) = 1 - \left(\frac{K}{\bar{K}}\right)^\alpha$ where $\alpha > 0$ is a shape parameter. This distribution features several properties that are salient in the cross-section of firms. In particular, it captures the notion that most of debt issuance is done by a tail of very large firms with low per-unit costs $K_i$.\footnote{The size distribution of firms is fat-tailed and well-described by a Pareto form (Gabaix 2011, Chaney 2018). In addition, we expect foreign currency issuance fixed costs (Maggiori et al. 2020) to give rise to a negative correlation between costs paid per unit of debt and firm size.} These large firms will be the first to sort endogenously into foreign currency issuance, and therefore small increases in the threshold $\bar{K}$ in the neighborhood of the lower boundary $\underline{K}$ will give rise to disproportionately large movements in the masses $M$, as compared to similar increases in $K$ at higher levels of entry.\footnote{Appendix Section A.3 formalizes the conditions required for equilibrium existence under a broader class of distributions, as well as the second-order conditions associated with the model.}

We begin by studying the class of equilibria in which firms in country $B$ switch to currency $A$ (class BA equilibria). Firms in country $B$ play the threshold strategy described in Lemma 3, and all firms in country $A$ issue in home currency. The marginal firm in country $B$ (one for which $K_i = \bar{K}$) is indifferent between issuing in the two currencies. Therefore an interior equilibrium threshold $\hat{K}$ satisfies the following indifference condition, which is a
specialization of equation (22):\(^{12}\)

\[
\begin{align*}
\lambda_A \left[ m_{F,A} + \phi m_{I,A} \right] - \hat{K} & = \lambda_B \left[ m_{F,B} + \phi m_{I,B} \right], \\
\bar{U}_{B\rightarrow A}: \text{Utility from issuing in foreign currency} & = \bar{U}_{B\rightarrow B}: \text{Utility from issuing in home currency}
\end{align*}
\]  

(25)

To obtain the masses of buyers and sellers, we also specialize the market clearing conditions (20) and (21) to this class of equilibria. The masses of liquidity demanders (buyers) in the two currencies are

\[
m_{F,A} = \phi \left[ F_A + H(\hat{K}) F_B \right], \quad m_{F,B} = \phi \left( 1 - H(\hat{K}) \right) F_B,
\]  

(26)

while the masses of liquidity suppliers (sellers) are

\[
m_{I,A} = G_A + F_A + H(\hat{K}) F_B, \quad m_{I,B} = G_B + \left( 1 - H(\hat{K}) \right) F_B.
\]  

(27)

Figure 2a plots the curves \(\bar{U}_{B\rightarrow A}\), and \(\bar{U}_{B\rightarrow B}\) as functions of the threshold cost \(\hat{K}\). These curves capture the expected utility of the marginal firm \((K_i = \hat{K})\) from issuing in foreign currency or home currency. The figure continues to keep the country fundamentals symmetric \((\lambda_j, G_j, F_j)\). The shapes of these two curves reflect the economic forces at work. The curve \(\bar{U}_{B\rightarrow B}\) is monotonically decreasing since higher values of \(K\) correspond to higher entry into currency \(A\), which reduces the thickness of currency \(B\) markets, lowering the utility of home currency issuance. Conversely, higher entry raises the expected utility of foreign currency issuance, which is a force pushing \(\bar{U}_{B\rightarrow A}\) higher. The curve \(\bar{U}_{B\rightarrow A}\) is however also subject to a second force, since as \(\hat{K}\) increases, the identity of the marginal firm changes: it is now a firm with higher foreign issuance cost \(K_i\). This gives rise to a linearly decreasing component of \(\bar{U}_{B\rightarrow A}\). Given the Pareto distribution of costs, there is a diminishing marginal impact of entry from increasing the threshold cost \(\hat{K}\), which mediates the relative strength of the forces impacting \(\bar{U}_{B\rightarrow A}\), giving rise to its non-monotonic (concave) shape.

The model features three equilibria of class \(BA\). Two equilibrium points (labeled 1 and 2, and occurring at \(\hat{K}_1\) and \(\hat{K}_2\), respectively) lie at the intersections of the two curves \(\bar{U}_{B\rightarrow A}\) and \(\bar{U}_{B\rightarrow B}\). At these intersections, the expected utility from issuing in \(A\) and \(B\) is equalized for the marginal firm with threshold cost \(K = \hat{K}\). The interior equilibrium featuring low entry (point 1) is unstable while the high-entry equilibrium (point 2), featuring currency dominance, is stable. A further equilibrium point (labeled 0) is at \(\hat{K} = K\). In this

\(^{12}\)For ease of exposition, here we do not separately write out the condition for equilibria at the left boundary \(\hat{K} = K\), although these are of course valid if the complementary slackness condition in the second row of equation (22) is satisfied.
equilibrium, no firms issue in foreign currency: this equilibrium is now stable (relative to the $K_i = 0$ case) because of the presence of fixed costs $K_i > 0$, which make $\bar{U}_{B \to B}$ higher than $\bar{U}_{B \to A}$ in the neighborhood of $\bar{K} = \bar{K}$.

As also seen in Proposition 1, there is a second, symmetric class of equilibria as well. These are equilibria in which firms in country $A$ issue in foreign currency, while all firms in country $B$ remain in home currency (class $AB$ equilibria). For this class of equilibria, $\bar{K}$ now characterizes the threshold strategy played by firms in country $A$. The analysis is analogous. Since it is now the marginal firm in country $A$ that needs to be indifferent, the indifference condition determining an interior equilibrium threshold $\hat{K}$ in this case is:

$$
\lambda_B [m_{F,B} + \phi m_{I,B}] - \hat{K} = \lambda_A [m_{F,A} + \phi m_{I,A}],
$$

and the masses $\mathcal{M}$ are obtained by performing the analogous specialization of (20) and (21).

Figure 2b provides a graphical analysis by showing the curves $\bar{U}_{A \to B}$, and $\bar{U}_{A \to A}$ as functions of the threshold $\bar{K}$, continuing to keep country fundamentals symmetric. As in the previous case, there are two interior equilibria: a low-entry unstable one (labeled 3) and a high-entry stable one (labeled 4). The equilibrium in which no firm switches (at $\bar{K} = \bar{K}$, labeled 0) also features. This is because the no-entry equilibrium has two isomorphic representations: one as a class $BA$ equilibrium, and one as a class $AB$ equilibrium.\textsuperscript{13}

We note that despite the presence of currency dominance, our model with positive foreign issuance costs rules out winner-takes-all equilibria in which only one currency survives, and the dominant equilibria remain interior. Hence in this respect, our model is consistent with the evidence in Eichengreen et al. (2017) that despite currency dominance, multiple currencies co-exist as units of denomination in equilibrium.

Summing up, the model in this case—with symmetric fundamentals but heterogeneous costs $K_i$—features a total of five equilibria, only three of which are stable. The three stable equilibria are the no-entry equilibrium (point 0), the equilibrium featuring high entry from country $A$ firms into currency $B$ (point 2), and the equilibrium featuring high entry from country $B$ intro currency $A$ (point 4).

\textsuperscript{13}If writing down the threshold strategy from the perspective of firms in country $B$ (class $BA$ representation), the no-entry equilibrium corresponds to the case in which $\bar{U}_{B \to B}(\bar{K}) \geq \bar{U}_{B \to A}(\bar{K})$. If expressing the strategies from the perspective of firms in country $A$ (class $AB$ representation), it corresponds to the case in which $\bar{U}_{A \to A}(\bar{K}) \geq \bar{U}_{A \to B}(\bar{K})$. In either case, each condition independently implies by Lemma 2 that all firms in both countries choose to denominate in home currency.
Figure 2: Characterizing equilibria in the heterogenous cost case

Notes: Panel A plots the expected utility of a firm in country B switching to issuing in foreign currency ($\bar{U}_{B\to A}$), and of a firm in B issuing in home currency ($\bar{U}_{B\to B}$), as a function of the threshold cost $\bar{K}$. Panel B plots the expected utility of a firm in country A switching to issuing in foreign currency ($\bar{U}_{A\to B}$) or issuing in home currency ($\bar{U}_{A\to A}$), also as a function of $\bar{K}$. The graph is for the case of symmetric fundamentals ($\lambda_j, G_j, F_j$) and positive currency-switching costs $K_i$ distributed over $[K, \infty)$ with a Pareto CDF $H(K)$. There are five equilibria.

3.3 Asymmetries in Country Fundamentals

We can now consider comparative statics with respect to country fundamentals, hence introducing asymmetry in fundamentals between the two countries $A$ and $B$. The following proposition characterizes these comparative-statics results.

Proposition 2. In the case with positive $K_i \in [K, \infty)$, the following holds, respectively for class BA and class AB equilibria:

1. **Class BA equilibria:** An increase in country A’s government bond supply $G_A$ increases foreign-currency issuance by B-firms ($\frac{\partial \bar{K}}{\partial G_A} > 0$) in the stable high-entry equilibrium (point 2 in Figure 2). A sufficiently high increase in $G_A$ dissolves the no-entry and low-entry equilibria (points 0 and 1), resolving equilibrium multiplicity in favor of the high-entry equilibrium. The same holds for increases in the overall matching intensity $\lambda_A$ and increases in the mass $F_A$ of firms in country $A$.

2. **Class AB equilibria:** An increase in country A’s government bond supply $G_A$ decreases foreign-currency issuance by A-firms ($\frac{\partial \bar{K}}{\partial G_A} < 0$) in the stable high-entry equilibrium (point 4 in Figure 2). A sufficiently high increase in $G_A$ dissolves all class AB equilibria, resolving equilibrium multiplicity in favor of the high-entry class BA equilibrium point. The same holds for increases in the overall matching intensity $\lambda_A$. Increases in the mass $F_A$ of firms in country A have ambiguous impact on entry, with the direction
Figures 3a and 3b provide a graphical exposition of these comparative statics results. Consider first an increase in government bond supply $G_A$. As shown in Figure 3a, this acts as an outward shift of the blue curves $\bar{U}_{B \rightarrow A}$ and $\bar{U}_{A \rightarrow A}$: as government bond supply grows, the mass of investors $m_{I,A}$ that are sellers in the $A$ money markets increases, which in turn benefits issuers in currency $A$'s market as it expands the pool of liquidity available to them at date $t_1$. If this shift occurs starting from the high-entry class-BA equilibrium (point 2), the equilibrium threshold $\hat{K}$ shifts further to the right, which means entry is increasing, and currency $A$'s dominance is more entrenched. If the shift occurs starting from the high-entry class-AB equilibrium (point 4), $\hat{K}$ conversely shifts to the left, as the resulting asymmetry in fundamentals weakens currency $B$'s dominance. 

Crucially, increasing $G_A$ sufficiently will cause a qualitative shift in the configuration of equilibria in both equilibrium classes. First, a sufficient upward shift of the curve $\bar{U}_{A \rightarrow A}$ will raise $\bar{U}_{A \rightarrow A}$ above $\bar{U}_{A \rightarrow B}$ over the entire domain $[K, \infty)$ for class $AB$ equilibria so that the two curves never intersect. Intuitively, this implies that sufficiently strong asymmetry between $G_A$ and $G_B$ dissolves all class $AB$ equilibria in favor of class $BA$ equilibria. Simply put, equilibria in which $B$ is dominant cannot survive once country $A$ achieves a sufficiently large advantage in government debt supply, all other fundamentals equal.

Second, large enough increases in $G_A$ also impact the configuration of $BA$ equilibria themselves. This happens once the $\bar{U}_{B \rightarrow A}$ curve crosses above $\bar{U}_{B \rightarrow B}$ at $\hat{K} = K$. Once this threshold is crossed, the no-entry and low-entry equilibria (points 1 and 2) both disappear, leaving the high-entry equilibrium as the sole remaining one. These results illustrate another key economic point, which we discuss further below: multipolar equilibria (with denomination dispersed among multiple competitor currencies) can only survive in a world of roughly symmetrical fundamentals, while sharp asymmetries among countries bring about dominance.

Next, consider an increase in the mass of firms $F_A$ in country $A$, the effects of which are shown in Figure 3b. Growing the size of the private sector is not equivalent to increasing safe government debt supply: while these have the same effect from the perspective of class $BA$ equilibria, the same is not true for class $AB$ equilibria. Similarly to an increase in $G_A$, increasing $F_A$ shifts up the curve $\bar{U}_{A \rightarrow A}$, since a share of the additional mass of $A$ firms will continue to issue in home currency, improving the liquidity of the currency $A$ markets. On the other hand, some of the additional mass will go towards improving the liquidity of currency $B$ markets, in proportion to the share of $A$ firms that issue in foreign currency.
Figure 3: Comparative statics

(a) Increasing government bond supply $G_A$

(b) Increasing firm mass $F_A$

Notes: We consider introducing asymmetry in country fundamentals by increasing government bond supply $G_A$ (Panel A) or the mass of firms $F_A$ (Panel B).
For \( \bar{K} > \bar{K} \), increasing \( F_A \) therefore tilts up the \( \bar{U}_{A \to B} \) curve as well, which has the effect of increasing, rather than decreasing, entry of \( A \) firms into currency \( B \).

The net impact of increasing \( F_A \) on entry in class \( AB \) equilibria therefore depends on which of these two forces prevails. The relative strength of these two forces depends on the value of \( \bar{K} \): for high value of \( \bar{K} \), the second force will tend to be relatively stronger, and vice-versa. Figure 3b shows an example in which the starting value of \( \bar{K} \) in the stable \( AB \) equilibrium is sufficiently high that the second force is stronger, and as a result, the increase in \( F_A \) leads to overall higher entry, unlike in the case of increases in \( G_A \). However, it is possible for the net direction of the comparative static to take the opposite sign. This illustrates a fundamental difference between sovereign and private issuance in our model: if a country does not start out as a dominant currency issuer, simply growing the size of the private sector is not guaranteed to facilitate the internationalization of its currency, and in fact can be counterproductive. Increasing the stock of safe government debt supply is instead a more reliable instrument, as it will facilitate international usage of the currency regardless of the starting equilibrium conditions.

### 3.4 Complementarity Between Liquidity Provision and Demand

In the baseline set of international equilibria outlined in Section 3.2, firms trade off the costs of currency mismatch with both of the two benefits of dominant-currency issuance: favorable convenience yields and high settlement liquidity. In reality, individual corporate issuers in the cross-section of firms may benefit disproportionately from one or the other incentive. For example, global corporate borrowers with poor credit ratings may not be able to benefit from convenience yields, as default risk makes their debt not sufficiently money-like: nonetheless, these borrowers will still be attracted to issuing in the dominant currency because of its settlement liquidity. As such, these issuers act as net liquidity demanders in the dominant-currency money markets. On the other hand, particularly safe borrowers (for which we use the example of the German sovereign-backed issuer KfW) may act as net liquidity suppliers and be drawn to dominant currency issuance primarily to capture the convenience yield.

In this section, we present an extension of our baseline model that separates out these two roles by allowing issuers to be heterogeneous in their role as liquidity demanders or suppliers. Allowing for this further degree of heterogeneity in the cross-section allows us to formalize the complementary nature of the denomination decisions of these two types of corporate borrowers. Hence we provide a positive explanation for why, today and as well as historically, we observe a wide range of different borrowers issue debt in the dominant
currency—from the safest global borrowers all the way down to more speculative and risky ones. At the same time, we highlight how strategic complementarities between these various actors further reinforce the dominant currency equilibrium.

We focus on the class BA equilibria by separating the liquidity demand and liquidity provision roles in the cross-section of firms in country B. Specifically, we now allow the overall mass of firms $F_B$ to be composed of two different groups of firms:

- A first mass $F^+_B$ consists of pure liquidity suppliers: these are issuers for whom $\phi = 0$, which therefore never experience an early realization profits and hence have no motive for demanding liquidity. Given that $\phi = 0$ for these firms, they will not contribute to the liquidity-danner masses $m_{F,j}$ in either country.

- A second mass $F^-_B$ consists of pure liquidity demanders. These are firms whose bonds have no possibility of re-sale in the money market of date $t_1$, so that effectively $\lambda_j$ (which is now heterogenous for different assets) is zero for these firms’ issues. These firms therefore will not contribute to the liquidity-supplier masses $m_{I,j}$.

The cost $K_i$ follows the same distribution $H(K)$ in these two subgroups of firms, and $F_B = F^+_B + F^-_B$. These two groups of firms will now have two different endogenous equilibrium thresholds $(\hat{K}^+, \hat{K}^-)$: liquidity suppliers issue in foreign currency if and only if $K_i < \hat{K}^+$, while liquidity demanders issue in foreign currency if and only if $K_i < \hat{K}^-$.

We consider the equilibrium determination of $(\hat{K}^+, \hat{K}^-)$ in the DGP case, in which $\theta = 1$. The equilibrium conditions pinning down the two thresholds are

$$\lambda_A m_{F,A} - \hat{K}^+ = \lambda_B m_{F,B},$$

$$\lambda_A \phi m_{I,A} - \hat{K}^- = \lambda_B \phi m_{I,B}.\quad (29)$$

To complete the characterization of the equilibrium, the liquidity-demand masses in the two countries are

$$m_{F,A} = \phi \left[ F_A + H(\hat{K}^-) F^-_B \right], \quad m_{F,B} = \phi \left( 1 - H(\hat{K}^-) \right) F^-_B,\quad (31)$$

while the liquidity-supply masses are

$$m_{I,A} = F_A + G_A + H(\hat{K}^+) F^+_B, \quad m_{I,B} = G_B + \left( 1 - H(\hat{K}^+) \right) F^+_B.\quad (32)$$
The solution is characterized by a two-equation system for the equilibrium values of \((\hat{K}^+, \hat{K}^-)\):

\[
\hat{K}^+ = \phi \left[ \lambda_A F_A - \lambda_B F_B^- + (\lambda_A + \lambda_B)H(\hat{K}^-)F_B^- \right],
\]

\[
\hat{K}^- = \phi \left[ \lambda_A (F_A + G_A) - \lambda_B (G_B + F_B^+) + (\lambda_A + \lambda_B)H(\hat{K}^+)F_B^+ \right].
\]

Equations (33) and (34) express the best responses of liquidity suppliers and liquidity demanders, respectively, to the entry decisions of the other type of firm. Crucially, these coupled entry decisions are complementary in that entry by one class of firms increases the entry incentives of the other class and vice-versa, as formalized in the following proposition.

**Proposition 3.** The equilibrium entry decisions of liquidity suppliers \((F_A^+)\) and liquidity demanders \((F_B^-)\) are complementary, as their respective best responses satisfy:

\[
\frac{\partial \hat{K}^+(\hat{K}^-)}{\partial \hat{K}^-} > 0, \quad \frac{\partial \hat{K}^-(\hat{K}^+)}{\partial \hat{K}^-} > 0.
\]

Hence strategic complementarity in the denomination decisions of these two classes of borrowers reinforce the dominant-currency equilibrium.

**Proof.** See Appendix Section A.2.

Figure 4 provides a parametric plot of the best response functions (33) and (34) in the two-dimensional cost space of this extended model, continuing to use a Pareto distribution \(H(K)\) with symmetric country fundamentals. In the symmetric-fundamentals case, the system features two stable equilibria, one in which neither kind of firm enters, and one in which both kinds have strong entry.

When we introduce asymmetry by increasing \(G_A\), a sufficient rightward shift of the \(\hat{K}^-\) curve will also resolve equilibria multiplicity in favor again of the high-entry equilibrium, in which currency dominance is now further reinforced by these strategic complementarities.

### 3.5 Generalized Convenience Yields and Issuance Predictions

The analysis thus far has focused on presenting analytical results for the \(\theta = 1\) case, as in Duffie et al. (2005). Removing this restriction provides the following equilibrium condition that determines \(\hat{K}\), again focusing for ease of exposition on BA equilibria:

\[
\lambda_A (m_{F,A} m_{I,A})^{\theta-1} \left[ m_{F,A} + \phi m_{I,A} \right] - \hat{K} = \lambda_B (m_{F,B} m_{I,B})^{\theta-1} \left[ m_{F,B} + \phi m_{I,B} \right].
\]  

\(U_A\): Utility from issuing in foreign currency

\(U_B\): Utility from issuing in home currency
Figure 4: Illustrating complementarity in the choices of liquidity demanders and liquidity suppliers

Notes: This figure plots the best-response functions for the coupled entry decisions of liquidity suppliers ($\hat{K}^+$) and liquidity demanders ($\hat{K}^-$) in the extended model of Section 3.4. These are shown in the cost space ($\hat{K}^+, \hat{K}^-$) for the case of symmetric country fundamentals.

The market clearing expressions that characterize the masses $\mathcal{M}$ remain unchanged. This more general set of equilibrium conditions yields economic results that are equivalent to those studied in the baseline case, but it captures a richer behavior of asset prices in response to variations in bond supply.\(^\text{14}\)

A key difference between the positive implications of the $\theta = 1$ case and those of the general $\theta < 1$ case concerns the mechanism of transmission of asset supply to convenience yields. Recall the expression for the convenience yield in country $A$, which is

\[
P_{0,A} - \beta^2 = \frac{\lambda_A \beta (1 - \beta)}{2} m_{F,A} m_{I,A}^{\theta - 1}.
\]  

(37)

In the general case ($\theta < 1$), both liquidity demand ($m_{F,A}$) and liquidity supply ($m_{I,A}$) impact the convenience yield: intuitively, rising liquidity demand increases the convenience yield, while rising liquidity supply reduces it. Figure 5a illustrates this point for a parametric case where $\theta = 0.9$: while entry is increasing in sovereign debt supply $G_A$, the convenience yield is decreasing in $G_A$. This prediction matches the empirical findings in Krishnamurthy and Vissing-Jorgensen (2012) that convenience yields are decreasing in government debt supplies.

\(^{14}\)In the $\theta < 1$ case, we need to make assumptions on the vector of parameters in order for the problem to be well-behaved and the equilibrium configuration to be equivalent to the one we present in Section 3.2. For example, we need $\theta$ to exceed a critical value $\theta^*(\Theta) \in (\frac{1}{2}, 1]$ which depends on parameters.
(a) Case 1: Convenience yield decreasing in $G_A$

(b) Case 2: Convenience yield increasing in $G_A$

**Notes:** We focus on equilibria of the class $BA$, in which $B$ firms switch to currency $A$. We show the behavior of the convenience yield $P_{0,A} - \beta^2$ for currency $A$ as a function of government debt supply $G_A$. The analyses show these simulated comparative statics selecting the stable equilibrium with maximum entry threshold, $\hat{K}_{\text{max}}$, which is also plotted. Panel A uses $\theta = .9$, while Panel B uses $\theta = 1$. The rest of the parameter values in this example are $F_A, G_A, G_B, \lambda_A, \lambda_B = 1; \phi, K = .5; \alpha = 1.5; \text{ and } F_B = .1$, where we set $F_B$ low to emphasize the impact of liquidity supply on convenience yields, minimizing the indirect entry effect.

We illustrate this crowding out mechanism in the model in the context of the extended model of Section 3.4, in which we separate the liquidity provision and liquidity demand roles of firms. With this additional form of firm heterogeneity, we can show that it is the liquidity-supplying safe borrowers that are crowded out by government issuance, since they are the issuers that benefit from higher convenience yields. Thus, while higher government debt supply still on net leads to increased entry, this is partially offset by the impact on asset prices, lowering convenience yields, and crowding out private liquidity suppliers.

We show the differential impact of government debt issuance on the entry decisions of these two types of firms in Figure 6 with a parametric example. The figure plots the entry thresholds of liquidity demanders ($\hat{K}^-$) and liquidity suppliers ($\hat{K}^+$) as we vary $G_A$ with $\theta = .9$, keeping the other parameters the same as in Figure 5a. The equilibrium conditions in this case, which are generalized counterparts to those in equations (29) and (30), become:

\begin{align*}
\lambda_A m_{F,A}^\theta m_{I,A}^{\theta-1} - \hat{K}^- &= \lambda_B m_{F,B}^\theta m_{I,B}^{\theta-1}, \quad (38) \\
\lambda_A \phi m_{F,A}^{\theta-1} m_{I,A}^\theta - \hat{K}^- &= \lambda_B \phi m_{F,B}^{\theta-1} m_{I,B}^\theta. \quad (39)
\end{align*}

Entry by risky firms (the liquidity demanders) increases with $G_A$, since these firms benefit by
the improved liquidity of A markets but do not care about convenience yields. On the other hand, entry by safe firms (the liquidity suppliers) falls with \( G_A \), since these firms do not value the settlement liquidity benefits, but rather only care about the declining convenience yield. We therefore have the simultaneous crowding out of safe borrowers and crowding in of risky borrowers.

Figure 6: Crowding in and crowding out in the model with further heterogeneity

Notes: We consider comparative statics for the model with additional firm heterogeneity featured in Section 3.4. We plot the entry thresholds for liquidity suppliers (\( \hat{K}^+ \)) and liquidity demanders (\( \hat{K}^- \)) as a function of government debt supply \( G_A \). The analyses show these simulated comparative statics selecting the equilibrium with highest entry. We use the same parameters in Figure 5a, with \( \theta = .9 \). We set \( F_B^+ = F_B^- = .05 \), so that the total mass of B firms is equal to the one used in Figure 5.

In featuring both of these mechanisms, our paper differs from Gopinath and Stein (2021) and Chahrour and Valchev (2022). In the former paper, more supply of dollar-denominated US government debt hampers the dollar dominance equilibrium and reduces dollar issuance, so it crowds out private sector entry. In the latter, higher US sovereign supply crowds in firms’ dollar issuance. Our theory highlights how both crowding out and crowding in can occur at the same time, but for different types of issuers. This of course all holds within a given equilibrium: as noted earlier, increasing \( G_A \) sufficiently can trigger an equilibrium transition, towards a dominance equilibrium.

The \( \theta = 1 \) case is a knife-edge parameter configuration in which the liquidity supply channel is shut off: in this special case, shifts in the liquidity supply schedule (such as changes in government debt supply \( G_A \)) do not have a direct effect on bond prices, but rather only impact them through their indirect effect on entry. In the \( \theta = 1 \) case, the convenience yield is therefore unambiguously increasing in government debt supply \( G_A \), and this effect is entirely
mediated through stronger equilibrium entry \( \hat{K} \). Figure 5b illustrates this case where both the convenience yield (in blue) and entry, \( \hat{K}_{\text{max}} \) (in orange), are increasing functions of \( G_A \).

4 Liquidity and Denomination Throughout History

Over the course of the last four centuries of the international monetary system, the dominant currency in global finance has been the Dutch florin followed by the British pound sterling and then the US dollar. We trace these transitions to large asymmetries in the supply of safe government-backed debt. We then show that the sovereign’s ability and incentives to invest in institutions that deepen financial market liquidity are larger in the center country—the one which hosts the dominant currency. This generates additional complementarities that deepen the dominance regime. We discuss the role of international trade throughout these regimes, which also introduces additional complementarity to dominance in finance. Finally, we discuss welfare in the context of Bretton Woods era liquidity coordination and the impact of modern policy tools such as central bank swap lines in the context of aggregate risk.

4.1 Transitions Throughout History

Emergence of the First Global Currency. Prior to the Dutch florin era in the 17th and 18th centuries, the European financial landscape was markedly different. However, the Italian city-states during the Renaissance were the most similar to Amsterdam in becoming prominent in both trade and finance. Yet the Republics of Genoa, Venice, and Florence conducted their finance in a constellation of local currencies, such as the Genoese and Venetian lira and the Florentine florin, and none achieved the centrality that the Dutch florin would attain in subsequent centuries.

Our model rationalizes this difference between the Italian city-states and the later Dutch experience and attributes it to the economic mechanisms centered on increasing returns to scale in liquidity provision. The multipolar equilibrium of the multiple Italian currencies corresponds to equilibrium point 0 of Figure 2, which features multiple currencies of roughly equal importance. This multipolar equilibrium is stable as long as these currencies have separate and approximately symmetric underlying liquidity pools (as determined by \( G_j \)).

We consider this to be an accurate representation of the historical context in which the constellation of Italian currencies circulated in the form of physical coins. They were in fluctuating and uncertain supply in any given place and time because they were often transported, debased, or re-minted. Indeed, Italy during the Renaissance faced the same
settlement frictions stemming from a large variety of metallic coins that Amsterdam faced later on. However, while these city-states had large banks (including the Bank of San Giorgio dating from 1407 in Genoa and the Medici Bank from 1397 in Florence), none of them invested in creating a large and steady supply of safe debt in a common currency \( (G_j) \) as Amsterdam did.\(^{15}\) The Genoese lira and the Florentine florin therefore remained unremarkable in terms of their liquidity properties like the other coins of the era.

In contrast, the Dutch florin was a pure unit of account that lived on the ledgers of the Bank of Amsterdam and therefore did not suffer any of the usage frictions of metallic coin. A florin was a safe claim issued by the Bank of Amsterdam and backed by the City of Amsterdam. All bills of exchange settled in Amsterdam were required to be denominated in florin, so the initial supply was sufficient to settle all of Amsterdam’s liquidity demand. In the context of our model, the financial innovations by the Bank of Amsterdam allowed it to increase florin-denominated \( G \) sufficiently to trigger a shift from the multipolar equilibrium to unipolar dominance. It did so by pooling disparate assets (such as the various kinds of specie and coins that were used prior to the florin in trade) to back a single, unified money market in the florin unit of account. Moreover, the fact that the Dutch cities did not all compete with each others’ currencies, but rather followed Amsterdam’s lead with its florin, provided liquidity agglomeration that the Italian city-states lacked.\(^{16}\) In Section 4.3, we discuss Amsterdam’s incentives to invest in the florin technology in the first place.

The usefulness of the florin could also be measured in part by the \textit{agio}, the market exchange rate between locally circulating physical coin (current guilders) and bank florin. The \textit{agio} was steadily around 4.5 to 5% for most of the 17th and 18th centuries (Van Dillen, 1934, p. 91, 102), a premium that in part captured the florin’s superior liquidity.

\textbf{Pound Sterling Dominance.} The Bank of Amsterdam weakened considerably after the Fourth Anglo-Dutch War and eventually collapsed following the French invasion of 1795, during which concerns about the florin’s backing were reflected in a drop in the \textit{agio} to -14%.\(^{17}\) In our model, this is equivalent to a large and sudden contraction in \( G_A \) that shifts in the \( U_A \) curve and dissolves the dominant equilibrium of entry into \( A \). The period that followed without a clear dominant currency was resolved by Great Britain’s victory in the

\(^{15}\)Unlike Amsterdam, the Italian banks were slow to adopt ledgers and payments by account transfers. For instance, it was not until 1675 that the Bank of San Giorgio in Genoa issued depositors transferable vouchers reflecting deposit accounts (Willis, 1943, p. 12).

\(^{16}\)See Appendix B.1 for more details on the arrangement between Rotterdam and Amsterdam.

\(^{17}\)The Bank had secretly made large unsecured loans to the Dutch East India Company after the war, which had suffered large losses due to the naval blockade in the war. When these loans and losses came to light, the Bank suffered a run from which it never recovered.
Napoleonic Wars. The UK at that time had the largest economy and a government debt in pound sterlings of approximately 200% of GDP, which was coupled with a state capacity for taxation and credible governance that made this debt safe. As in the case of Amsterdam before it, having the largest pool of safe liquid assets for settlement initiated the process of dominance, which Great Britain held until the mid-20th century.

The closest likely competitor to British dominance was the French franc. However, France experienced several disruptions to its supply of safe government debt during the regime changes in the late 18th century, and its loss at Waterloo reduced its subsequent ability to float a large volume. Unlike Great Britain, it was required to pay off its wartime debt with much higher taxes instead of new long-term issuances, which were also at higher rates (Bordo and White, 1991). While the franc itself returned to the gold standard and maintained a stable value, the outstanding amounts of other forms of franc-denominated claims were much smaller than that of the pound’s. One illustration of pound dominance is that French firms were the largest foreign contributors to the corporate bond issuance in London, accounting for 25% on average from the mid-19th century to WWI.\textsuperscript{18}

**Dollar Dominance.** Following WWI, Eichengreen and Flandreau (2009) shows that the pound sterling and US dollar were held in roughly equal proportion by central banks as foreign exchange reserves throughout the interwar period. They note that the dollar overtook the pound shortly after WWI (consistent with dollar liquidity growing following the centralization of US financial markets and a large war-time debt), but that its status also shrank after the 1933 dollar devaluation (consistent with a collapse in confidence in the US government’s commitment to its currency).

During the Bretton Woods era immediately after WWII, the dollar’s dominant status was a result of the institutional centrality of the dollar within the gold exchange standard. Afterwards, the dollar’s supremacy grew because the source of commitment shifted from the US gold reserves to its fiscal capacity, the latter of which was much larger. This increase in fiscally-backed commitment solidified the dollar’s dominance even further.

The emergence of the Euro in the late 20th century similarly reflects a consolidation of commitment, represented by \( G \), across multiple countries into a single denomination. Similarly, the contraction of the Euro’s global usage after 2008 matches a decrease in \( G \), reflecting new information that government debt from countries like Spain, Italy, Portugal, and Greece did not contribute to the commitment backing the Euro.

\textsuperscript{18}Authors’ calculations based on bond data from the \emph{Investor’s Monthly Manual}. 

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Throughout History. Throughout history, there have also been numerous examples of countries whose economic size alone was not sufficient to propel them to dominance. For example, Spain was the largest and wealthiest global economy during the 17th century, and yet the much smaller City of Amsterdam emerged as the center of the international financial system. During the late 19th century, the US overtook Great Britain as the largest economy in the world. The projections now are that China will overtake US GDP in the next decade. Yet in all cases, within the language of our theoretical framework, these are examples of countries with large $F_j$ but small $G_j$. Spain lacked the government commitment and financial technology to create a liquid money market; the US financial sector and monetary system was fragmented until 1913; and China lacks both an internationally tradable base of government-backed money market instruments and confidence in its government’s commitment not to appropriate those assets.

4.2 Endogenous Investment in Financial Market Liquidity

Following the establishment of a dominant currency, the government of the dominant country has further incentives to innovate in institutions and financial technologies that advantage its market liquidity. These innovations are less costly to finance in the dominant currency, and they further entrench dominance. The history of the Bank of Amsterdam’s innovations in the florin, and the institutional changes in the London money market that impacted the pound sterling includes several dimensions of these additional sources of complementarity.

We first specify the sovereigns’ objective functions. The government of country $j$ maximizes the following:

$$W_j = F_j \int u_{i,j}^F(K_i) \, dH(K_i) + G_j(P_{0,j} - \beta^2).$$  \hspace{1cm} (40)

The first term in this objective function corresponds to the purely utilitarian welfare criterion that aggregates the preferences of domestic firms. In addition to this standard utilitarian objective, we allow the government to have a profit motive, which is reflected in the second term of equation (40). These seigniorage profits correspond to the revenues earned from the convenience yield on sovereign debt issued at $t_0$, which scales linearly with the size of government issuance $G_j$. These revenues could also be used to finance a costly investment in government-backed institutions such as ones that expand state or fiscal capacity.

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Note that investors always break even in equilibrium, so that we omit the utility of investors in writing the sovereign objective.
For the rest of the analysis in the paper, we consider the case in which \( A \) is the higher liquidity market, by imposing the parameter restriction that \( G_A > G_B, \lambda_A > \lambda_B, \) and \( F_A > F_B \). We therefore focus on class \( BA \) equilibria—the only ones that remain present once asymmetry in fundamentals is sufficiently large. The analysis is of course symmetric for the case in which \( B \) is the higher-liquidity country. Integrating firm preferences given the equilibrium masses \( \mathcal{M} \) then yields the following expressions for welfare in the two countries.

**Proposition 4.** For \( \theta = 1 \), up to an affine scaling the objective for the leading country (\( A \)) reduces to

\[
W_A = \lambda_A [G_A m_{F,A} + F_A (m_{F,A} + \phi m_{I,A})],
\]

while the objective for the follower country (\( B \)), again up to an affine scaling, becomes

\[
W_B = G_B \lambda_B m_{F,B} + F_B \lambda_B (1 - H(\hat{K})) (m_{F,B} + \phi m_{I,B}) + U_{B \rightarrow A},
\]

where \( U_{B \rightarrow A} \) is the utility of the \( B \)-firms that issue in foreign currency, given by

\[
U_{B \rightarrow A} = F_B H(\hat{K}) \lambda_A (m_{F,A} + \phi m_{I,A}) - F_B \int_{\hat{K}}^{K} Kh(K)dK.
\]

The expressions for the two countries’ government objectives derived in Proposition 4 reveal several aspects of the structure of sovereign incentives in this environment. Consider the government’s incentives to invest in commitment technologies that allow it to expand \( G \). For example, investing in the military strengthens the government’s access to resources, thereby expanding fiscal capacity. First, the government’s ability to invest is higher in the dominant country because the convenience yields that the government earns from issuing safe debt are themselves a revenue stream that can reinforce the government commitment (Jiang et al., 2020b). Second, the government incentives to expand its safe debt issuance correspond to \( \frac{\partial W_j}{\partial G_j} \), which is also higher for the leader country (\( A \)) than the follower country.\(^{20}\)

This initial difference in the ability and incentives to invest furthermore impacts an additional complementarity: the incentive \( \frac{\partial W_j}{\partial G_j} \) is increasing with entry into \( A \)'s markets (\( \hat{K} \)), and investments in commitment further spur entry, leading to an endogenous rise in \( \hat{K} \), as formalized in the following proposition.

**Proposition 5.** The sovereign’s incentives to invest in government commitment are larger in the dominant country, and government commitment is complementary to issuance in the

\(^{20}\)The incentive to expand \( G_j \) is increasing in measures of dominance (\( \hat{K} \)), private firm size (\( F_j \)), financial technologies (\( \lambda_j \)), and in the degree of the private sector’s demand for liquidity (\( \phi \)).
Proof. This follows from differentiating $W_A$. 

During the period of pound dominance, the British government issued debt at the most advantageous terms of any sovereign, which gave it a funding advantage for military and naval expansion. These investments enabled further imperial expansion that increased its tax base and access to resources, thereby expanding fiscal capacity to issue more debt. Additional debt issuance then deepened the asymmetry of Britain’s financial market depth relative to other countries’, drawing in more firms into the pound equilibrium and lowering the costs of financing future investment. This positive dynamic feedback loop leads to the endogenous persistence of dominant regimes.

We next consider the case of innovations that increase the pledgeability of private revenues. As we discuss in the next section, financial development has taken the form of increases in government safe asset supply as well as improvements in the capacity of the private sector to issue safe assets. We extend the model to include a country-specific pledgeability parameter $\rho_j$. When firms apply for a bond issuance, they find out whether their project revenues are fully pledgeable (probability $\rho_j$) or non-pledgeable (probability $1 - \rho_j$). The borrowing ability is idiosyncratic, so that *ex ante* a given firm is able to pledge its revenues with probability $\rho_j$, and the law of large numbers applies across firms. After firms decide the currency in which to issue their debt, they find out their pledgeability and if they choose to borrow they incur the fixed cost $K_i$, as in the basic model. Thus, $\rho_j$ captures the pledgeability of firm revenues in country $j$.

The expected utility of borrowing in country $j$ is then proportional to

$$\rho_j \lambda_j [m_{F,j} + \phi m_{I,j}], \quad (45)$$

and the equilibrium condition for the marginal firm is now:

$$\rho_A \left[ \lambda_A (m_{F,A} + \phi m_{I,A}) - \hat{K} \right] = \rho_B \left[ \lambda_B (m_{F,B} + \phi m_{I,B}) \right]. \quad (46)$$

Increasing $\rho_A$ increases the benefits of issuing in currency $A$ in equation (46), thereby requiring the equilibrium $\hat{K}$ to adjust to a higher value. As in the case with the government’s commitment technology, the incentive to invest in firm pledgeability is increasing with size.
and dominance, which we formalize in the following proposition:

**Proposition 6.** The sovereign’s incentives to invest in firm pledgeability are larger in the dominant country, and firm pledgeability is complementary to issuance in the dominant currency:

\[
\frac{\partial W_A}{\partial \rho_A} > \frac{\partial W_B}{\partial \rho_B}, \quad \frac{\partial^2 W_A}{\partial \rho_A \partial \hat{K}} > 0, \quad \frac{\partial \hat{K}}{\partial \rho_A} > 0.
\] (47)

**Proof.** See Appendix Section A.2.

Equipped with the theoretical result above, we now discuss several ways in which this interaction between currency dominance and incentives to invest and financially innovate have played out through history.

### 4.2.1 Investments in Commitment Technologies Throughout History

**Innovations and Complementarities in Amsterdam.** The Bank’s introduction of a specie-florin repo facility in 1683 is credited with propelling the florin to global prominence (Quinn and Roberds, 2014a). Before this facility, obtaining florin required depositing specific coins, and accounts were primarily held by those with liquidity needs in florin-denominated bills of exchange.

The specie repo facility was an investment in financial technology that greatly expanded access to florins by allowing individuals and businesses to monetize safe but illiquid assets. In the context of the model, the trust that the collateral posted in the repo facility would not be appropriated by the Bank of Amsterdam was key to its success, thereby generating \( G_A \) and deepening the florin equilibrium.

The popularity of this facility, which doubled the quantity of florins at the Bank of Amsterdam despite the fees charged to use it, also illustrates the complementarity in the issuance incentives that we model.\(^{21}\) For specie investors, issuing florin against their assets was profitable because of the convenience yield from the liquidity benefits of transacting in florin, reflecting the incentives to provide liquidity even by those without payment needs.

**Bank of England’s Changing Role.** In Great Britain, the evolving role of the Bank of England from its creation in 1693 to the 19th century encapsulates the institution’s changing incentives to facilitate financial market liquidity. At its founding, it was a private corporation

\(^{21}\)The quantity of florin doubled from approximately eight million to sixteen million from the mid 17th to the beginning of the 18th century and is credited with drawing Europe’s specie trade to Amsterdam, where it could be more profitably be conducted.
that was granted several privileges in return for raising and administering the Crown’s debt, and during the early part of its history, the Bank competed with other private banks such that it sometimes limited market liquidity in order to protect its own balance sheet.\textsuperscript{22}

Beginning in the 1830s, a series of legislative reforms changed the Bank of England’s role into one of a liquidity supplier. First, the Bank’s notes became legal tender for pound sterling debts in 1833, which legally expanded the supply of pounds sterling.\textsuperscript{23} Second, the entire note issuance was consolidated onto the Bank’s balance sheet and given a large fiduciary issue in 1844, which again expanded the supply of pound-denominated settlement instruments. Third, by the mid-19th century, the Bank of England established its role as a reliable lender of last resort to the financial sector (an innovation that Bagehot (1873) credits to the Bank), where its balance sheet explicitly became the backstop to the private bills market.

**Innovations and Complementarities in the London Money Market.** Institutionally, the private bills market also underwent changes. The legal codification of the contractual terms for bills of exchange coordinated the market on the terms of borrowing and the procedures for default, which reduced their information sensitivity (Dang et al., 2017) and collectively raised the safety and liquidity of the London money market. The growing clarity on the Bank of England’s discount window rules also helped to homogenize money market securities and raised the incentives to produce high quality “discountable” bills.

The government lowered the costs for the banking sector to create bills of exchange by deregulating private banks’ equity issuance in 1830. This fueled British banks’ overseas expansion, which took advantage of the growth in world trade during this period. London banks pursued a business model of issuing bills of exchange collateralized on the large base of international trade, which simultaneously allowed them to capture the dual benefits of issuance: the convenience yield for creating money-like assets in London and the settlement benefits from providing credit to exporters around the world (Xu, 2022). Their success relative to competitors from other nations was due to their access to the London bills market

\textsuperscript{22}The privileges restricted banking competition and gave the Bank of England a monopoly over note issuance. From 1697 until 1844, only the Bank could raise equity; all other banks were restricted to partnerships of six or fewer (after 1844, this was altered to a radius of 25 miles around London). In 1708, the Bank was granted an exemption to laws restricting bank note issuances to private partnerships (Broz and Grossman, 2004). The Bank’s abuse of its monopoly during the 1825 crisis led to the Banking Act of 1826, which mandated Treasury monitoring of small-denomination note issuance.

\textsuperscript{23}Like most currencies during this era, the pound sterling referred to a specific metallic coin, and obligations denominated in sterling were contracted to be repaid in those coins. However, coins were inconvenient for the reasons already discussed, and private banks like the Bank of England found it profitable to issue paper notes denominated in sterling (i.e., claims on sterling coin).
where they re-discounted the bills they underwrote. As in Amsterdam, these private and
government investments in commitment and pledgeability monetized a pool of previously
illiquid assets, further increasing market depth in the London money market.

**Innovations and Complementarities in the Dollar Market.** Innovation in the US
financial system has allowed privately-issued short-term debt instruments to add to the pool
of dollar-denominated money market liquidity. The growth of the US banking system and
of the commercial paper market (Greenwood and Scharfstein 2013) are early examples of
these liquidity-producing financial technologies. In more recent decades, securitization (Mian
and Sufi 2009, Keys et al. 2010) allowed for further private-sector production of safe liquid
assets, while the expansion of repo markets (Gorton and Metrick 2012, Krishnamurthy et
al. 2014) broadened liquidity in the overnight segment of dollar money markets. Our study
demonstrates that the financial and central banking advancements in the United States are a
natural consequence of the incentives that come with being the issuer of a dominant currency,
and that these developments further reinforce the currency’s dominance.

Correspondingly, the currency denomination choices of today’s worldwide debt issuers
feature the complementarities that we have illustrated through our theory. On the one
hand, very safe global issuers are attracted to the US dollar because of the convenience
yields attached to dollar-denominated assets, and they act as net liquidity suppliers to dollar
money markets. On the other hand, lower-rated entities all around the world also issue debt
in US dollars, drawn in by the ensuing liquidity benefits.

### 4.3 Finance and Trade

It has been clear both historically and today that trade and finance are intricately connected.
Historically, bills of exchange were issued first and foremost to finance international trade
transactions and today, trade finance continues to be an important part of the global credit
market. This section clarifies how trade intensity impacts the benefits from financial market
depth, and how trade invoicing impacts the forces we identify in our model. We then discuss
the history of trade and finance through the lens of our model.

#### 4.3.1 Role of Trade Intensity in Seeding Dominance

In our model, the parameter \( \phi \) captures the probability that a firm issuing debt at \( t_0 \) has a
liquidity need at \( t_1 \) because of a timing mismatch where revenues do not arrive at the same
time as the bond is due. For example, a merchant in Amsterdam can issue a bill of exchange
in order to buy goods at \( t_0 \) and face a \( \phi \) probability that the goods arrive early at \( t_1 \) before his bill is due at \( t_2 \). Historically, industries like international trade have had particularly high settlement liquidity needs: for example, for most of European and global history, it was difficult for merchants to predict exactly when their revenues would arrive.

We can capture the intensity of trade in the economy with variation in \( \phi_j \), such that \( \phi_A > \phi_B \) indicates that country \( A \) is more trade-intensive than country \( B \). The firm’s issuance benefits in equation (10) and the government’s welfare function in equation (40) are both increasing in \( \phi \). All else equal, the incentives to invest in institutions that expand \( G \) will be strictly higher in country \( A \) than in country \( B \).

In the case of Amsterdam’s history, its overall size, trade volumes, and military power were all subordinate to Spain’s. In fact, Spain’s access to resources suggests that it should have been able to sustain the largest pool of safe government debt at that time in history, but in fact it was a serial defaulter with low fiscal capacity. Through the lens of our model, trade and other finance-intensive industries were a small share of the Spanish economy (low \( \phi \)), which meant it had little incentive to invest in its financial market liquidity. In contrast, trade was the largest component of the Dutch economy, and investing in financial market depth therefore had proportionally much greater returns in Amsterdam.

4.3.2 Trade Invoicing in the Model

In our model, firms only receive revenues in their domestic currency. However, we can consider the case where firms receive a portion of their revenue stream to be denominated in foreign currency, as in the case of traded goods that are invoiced in dollars. For a given firm, this will reduce \( K_i \) because the receipt of some profits in the foreign currency reduces the amount of currency mismatch that a firm faces if it also issues dollar-denominated debt, as in our Appendix Section A.1.

A reduction in \( K_i \) for all \( i \) is a leftward shift of \( H(K) \), raising the utility of issuing abroad. Rewriting the equilibrium condition in equation (25) in terms of firm masses, the equilibrium condition is:

\[
\lambda_A \phi \left[ 2F_A + G_A + F_B H(\hat{K}) \right] - \hat{K} = \lambda_B \phi \left[ 2F_B + G_B + F_B(1 - H(\hat{K})) \right]
\]

Consider the case where there is an increase in the number of \( B \) firms receiving revenues in foreign currency. The cost distribution shifts left from \( H_0(K) \) to \( H_1(K) \) where \( H_0 \) f.o.s.d. \( H_1 \). This shift increases \( H(\hat{K}) \) and raises \( U_{B\rightarrow A} \) while reducing \( U_{B\rightarrow B} \) for the marginal firm.
at $\bar{K} = \hat{K}_0$. This benefit of entry will lead $B$ firms to issue abroad until a new equilibrium is reached that equates the two sides, which will occur at $\hat{K}_1 > \hat{K}_0$. Therefore invoicing in foreign currency leads to more financial dominance.

Separately, we can also consider how the denomination of trade invoicing might endogenously emerge in our model as a consequence of the liquidity forces discussed.\textsuperscript{24} Although this steps outside our formal analysis, if firms were also allowed to choose their currency of invoicing, dominant currency invoicing would emerge \textit{because} of the foreign exchange exposure firms have in their financial obligations. In that case, firms would choose to invoice in the foreign currency in order to reduce $K_i$, and the invoicing dominance is a byproduct of our equilibrium with financial dominance.

We note that our theory implies particular prescriptions for the paths that lead a currency to become globally dominant. For example, the theory suggests that in order to internationalize the renminbi, the Chinese government will need to open the capital account and invest in liquidity and financial innovation. In contrast, promoting trade invoicing in the renminbi would only spur debt market adoption to the extent that it reduces the cost of renminbi-denominated consumption.

### 4.3.3 Finance and Trade Through History

The preceding argument is that trade invoicing is a natural consequence of the liquidity forces of our model. While in the post-WWII dollar dominant world, the US has been both the largest country in international trade and had the deepest financial markets, history provides examples where volumes of trade invoicing was unlikely to have been the seed of currency dominance.

In the 17th and 18th centuries, the value of Spanish trade was a factor of two to six times larger than that of Dutch trade, and in fact, Dutch trade remained smaller throughout its centuries of florin dominance.\textsuperscript{25} The second global currency, the pound sterling, gained dominance in the 1820s during the first sovereign debt boom shortly after Waterloo, before the mid-century surge in world trade and expansion of the British Empire.\textsuperscript{26} Historically, world trade was also a much smaller component of world GDP: it was approximately 5%.

\textsuperscript{24}Goldberg and Tille (2008) and Gopinath et al. (2010, 2020) document the dominant role of the dollar in invoicing trade. Additional theoretical models of dollar use in trade invoicing include Rey (2001), Amiti et al. (2022), and Mukhin (2022).

\textsuperscript{25}In 1700, the ratio of Spanish to Dutch trade was just over two; by 1750 it had grown to almost six, and in 1800 it was still approximately five. See Appendix B.1.1 for the full sources and calculation.

\textsuperscript{26}As of 1815, the British Empire was still relatively small with little presence in Africa, the middle east, and Asia outside of India. Its land mass and population as a share of the world did not reach their peaks until the early 20th century with most gains in the second half of the 19th century.
Until the 1880s, at which point it grew to 10%.\textsuperscript{27}

In both the Amsterdam and British cases, there were investments in financial technologies by the state and private sector that enhanced liquidity. As we have explained, the Bank of Amsterdam’s ledger technology created liquidity in florin, which was the driving factor behind florin dominance in this period. For Britain, we described the legal and institutional developments around bills of exchange and the Bank of England that allowed international banks to securitize trade flows into forms of safe, liquid, money market instruments. During this period, British liquidity denominated in pounds underwrote 90% of world trade, while Britain in the goods market only comprised 50% of world trade (Xu, 2022).

In today’s dollar world, the volume of cross-border financial assets trade dwarfs the volume of real goods trade. Figure 7 plots the evolution of cross-border bank liabilities denominated in US dollars from 1975 until today relative to total world exports at a quarterly level. The average ratio in the last twenty years has been two. Accounting for the fraction of world trade that is denominated in US dollars (approximately 40% in recent years), the relative sizes are five-fold. Considering all debt securities in the world outstanding, the ratio of dollar-denominated financial claims to world trade annually has on average been eight-fold larger since 2015.

The relative volumes of financial assets and international trade in all currencies are

\textsuperscript{27}Following the interwar years, world trade did not regain this share of GDP until 1970, and it is now at approximately all-time highs of 25%.
similarly skewed. Annual global trade flows are approximately 24 trillion USD compared to global debt contracts outstanding of approximately 300 trillion USD. Assuming an average maturity of seven years and an average interest rate of 7%, annual debt payments are approximately 65 trillion USD, or three times global trade flows.

### 4.4 Liquidity Coordination and Welfare

At various points in history, there has been the recognition that one country may not be able to sustain the creation of enough liquid assets to support the pace of real economic growth. Triffin (1978) provided a prominent discussion of this concern in the context of gold-backed dollar reserves in the Bretton Woods period. Our model allows us to discuss liquidity provision policy and welfare from the perspective of a global planner. We continue to consider the case in which $A$ is the higher liquidity market, with the restriction that $G_A > G_B$, $\lambda_A > \lambda_B$, and $F_A > F_B$, and focusing on class $BA$ equilibria. We consider a planner whose objective is utilitarian over the preferences of the two governments:

$$W = W_A + W_B.$$  \hfill (49)

Our first result is that the planner’s choice of optimal entry features more entry into currency $A$ than the competitive equilibrium.

**Proposition 7.** Let $K^*$ be the value of $K \in [K, \infty)$ that maximizes global welfare $W$, and let $\hat{K}_{\text{max}}$ be the stable class $BA$ equilibrium point featuring highest entry.\(^{28}\) It holds that

$$K^* > \hat{K}_{\text{max}}.$$  \hfill (50)

**Proof.** See Appendix Section A.2.

At the heart of this result is the liquidity externality discussed in Section 2: entry into currency $A$ by firms in country $B$ carries social benefits in terms of improved market thickness, which are in excess of the private benefits of entry. Since these excess liquidity benefits are not internalized by the firms, there is too little private entry in equilibrium.

Conceptually, our setting presents an analogy with the theory of *natural monopoly* (Posner 1978). Here the higher-liquidity country ($A$) has aspects of a natural monopolist, since consolidating issuance in its currency is welfare-improving from a global perspective. An important difference between our model and theories of natural monopoly, however, is

\(^{28}\)This corresponds to point 2 in Figure 2.
that in this setting first-best equilibria are also interior, as in Eichengreen et al. (2017). The planner’s chosen equilibrium $K^\ast$ achieves the first best, which can always be made into a Pareto improvement relative to the private equilibrium $\hat{K}_{\text{max}}$ by introducing appropriate transfers.\footnote{We note that utility is transferable in our setting owing to the quasi-linear structure of preferences.} Similarly to the normative results that feature in theories of natural monopoly, optimal policy in this setting will therefore feature a subsidy to entry into currency $A$.

In the results above we have considered the entry problem from the perspective of the global planner. A second, related question is how the shadow value of increasing government debt supply $G_A$ in the leader country differs when viewed from the perspective of the global planner versus the sovereign in country $A$. That is, we are interested in comparing the two quantities $\frac{\partial W}{\partial G_A}$ and $\frac{\partial W_A}{\partial G_A}$. If the shadow value from the global planner’s perspective, $\frac{\partial W}{\partial G_A}$, is higher, then the planner will prefer to increase $G_A$ even beyond what is privately optimal for country $A$’s government, leaving open the possibility of welfare-improving international coordination in sovereign liquidity provision.

**Proposition 8.** The shadow value of increasing $G_A$ is higher from the global planner’s perspective, as compared to the perspective of the sovereign in the leading country ($A$), if and only if the following is satisfied:

\[
H(\hat{K})\frac{\lambda_A}{\lambda_B} > \frac{1}{2} \frac{G_B}{F_B} + [1 - H(\hat{K})] \iff \frac{\partial W}{\partial G_A} - \frac{\partial W_A}{\partial G_A} = \frac{\partial W_B}{\partial G_A} > 0. \tag{51}
\]

*Proof.* Differentiate (42) and (49) with respect to $G_A$, and rewrite to yield the inequality. \qed

In the expression above, note that if $A$ is dominant, then $H(\hat{K})$ is high, tending to one, while $1 - H(\hat{K})$ tends to zero, and likewise the ratio $\frac{\lambda_A}{\lambda_B}$ is high, increasing the likelihood that this condition is satisfied. The direction of this result therefore hinges on the relative magnitudes of $G_B$ and $F_B$. Improving liquidity in country $A$ has two effects on welfare in country $B$: on the one hand, it improves the utility of the infra-marginal $B$ firms that have already switched to foreign currency, but on the other hand it reduces the convenience yields earned on sovereign issuance $G_B$ by inducing stronger entry. If country $B$ has large private borrowing needs relative to the stock of safe government debt outstanding, the first effect outweighs the second, so that increasing $G_A$ is also welfare-improving from the perspective of country $B$.

If the condition in equation (51) is satisfied, the global planner will want to engineer incentives for country $A$ to further increase its liquidity supply, financing these with transfers from country $B$. These results provide a lens to interpret historical international liquidity
provision arrangements, such as the Bretton Woods agreements of 1944. In the Bretton Woods system, major world economies effectively coordinated on having liquidity provided by the United States with the bulk of the gold reserves underpinning the Bretton Woods gold standard held at the Federal Reserve.

Throughout the Bretton Woods period, the United States held more than 90 percent of the world’s gold reserves (Monnet and Puy 2020), with large transfers of gold from central banks outside of the United States to New York during and after WWII. These transfers and the resulting coordination on a US-backed gold convertibility system provide a historical counterpart to the possibility of welfare-improving coordination in international liquidity provision that features in our model. In response to the classic Triffin dilemma that the US gold reserves would be insufficient to back its internationally held liabilities, our model would have prescribed more liquidity coordination in the form of transfers of commitment (i.e., gold) to the United States.

4.5 Aggregate Risk, State-Contingent Liquidity, and Default Risk

Our discussion of liquidity provision did not draw a distinction between state-contingent and non-contingent expansions of liquidity supply $G_A$, although state contingency also played an important role within the Bretton Woods architecture—for instance, through the role of central bank swap line arrangements, which remain a core feature of the international monetary system to this day. A formal discussion of this topic requires extending the model to incorporate a role for aggregate risk, and hence we now suppose that $\phi$ is subject to an aggregate shock, realized at time $t_1$. The state is $\omega \in \Omega$, and in state $\omega$, which has probability $q_\omega$, the early profit realization probability $\phi$ takes on the value $\phi_\omega$. The state realization is a shock to aggregate liquidity demand: if the realized value of $\phi$ is higher, more firms experience timing mismatch and therefore there is more overall demand for liquidity.

We analyze this extended version of the model in the $\theta = 1$ case. We allow for state-contingency in the supply of government assets $G_A$ in the leading country, but we assume that $G_B, F_A$, and $F_B$ are all not state-contingent. The value of $G_A$ in each state $\omega$ is $G_A^\omega$. The following result presents the equilibrium indifference condition in this extended version of the model.

**Proposition 9.** In the model with aggregate risk, focusing on the case in which $A$ is the dominant currency (and hence considering class BA equilibria), the equilibrium condition
that determines the marginal firm \(i\) with \(K_i = \hat{K}\) is

\[
\lambda_A \left( \mathbb{E}[\phi_\omega] \left( 2(F_A + H(\hat{K})F_B) + \mathbb{E}[G^A_\omega] \right) + \text{Cov}[\phi_\omega, G^A_\omega] \right) - \hat{K} = \lambda_B \mathbb{E}[\phi_\omega] \left( 2(1 - H(\hat{K}))F_B + G_B \right).
\]  

(52)

Proof. See Appendix Section A.4.

With the addition of aggregate risk, the equilibrium condition is nearly the same as in the baseline case, except for the additional terms \(\mathbb{E}[G^A_\omega]\) and \(\text{Cov}[\phi_\omega, G^A_\omega]\), which are respectively the expected value of \(G^A\) and its covariance with the timing mismatch probability \(\phi_\omega\) across states. The intuition for the expectations term is straightforward: higher average government debt supply \(G^A_\omega\) shifts the left-hand side of this equality upwards and thereby increases the equilibrium entry threshold \(\hat{K}\).

The covariance term is positive when liquidity supply \(G^A_\omega\) increases in states of the world with high liquidity demand (and hence high \(\phi_\omega\)). When the covariance is zero, even with stochastic \(\phi_\omega\), the model collapses to the baseline case with constant \(\phi\). When \(G^A_\omega\) is positively correlated with \(\phi_\omega\), the extra covariance benefit makes currency \(A\) more attractive.

Central bank institutions such as the discount window and swap lines arrangements achieve positive covariance: both technologies allow the dominant currency issuer to expand liquidity supply when liquidity demand is particularly high.\(^{30}\) Swap lines provide emergency liquidity supply by one central bank to others: overwhelmingly, the central bank on the supplying end of these arrangements has been the US Federal Reserve, which provided US dollar liquidity in overseas markets during stress periods such as the global financial crisis of 2008–09 and the COVID crisis of 2020. Federal Reserve swap lines effectively promise a state-contingent expansion of dollar-denominated money market liquidity in times of need, hence increasing the expected liquidity benefits from issuing dollar debt. Hence we obtain the result that dollar swap lines reinforce the dollar dominance equilibrium and can be an important part of the architecture of the international monetary system.

The analysis also allows us to discuss default risk in government debt and the importance of the safety of the settlement asset. From the standpoint of \(t_0\), default risk is that \(G^A_\omega\) can fall in some states of the world. One effect of this risk is to reduce \(\mathbb{E}[G^A_\omega]\), thereby reducing government supply and entry \(\hat{K}\). A second effect is that if risk is higher in high-liquidity states of the world, then the covariance term is negative, further reducing effective

\(^{30}\)It is worth noting that the innovation of a central bank acting as a lender of last resort occurred in London during the period of British pound dominance while the latter has been an innovation of the Federal Reserve system during dollar dominance.
government supply and $\dot{K}$. Hence higher perceptions of sovereign default risk hinder a currency’s internationalization.

These effects concern risk realized between $t_0$ and $t_1$. There is a third effect that concerns risk realized between $t_1$ and $t_2$ that does not arise in the modeling of this section, but is likely an important concern. Bonds purchased at $t_1$ as settlement instrument for private debt at $t_2$ are poor settlement instruments if these bonds carry default risk. For example, if between $t_1$ and $t_2$, the bond defaults completely, then the firm will not own an asset to extinguish its debt. At $t_0$, default risk in government debt will mean that the currency is a less attractive currency in which to denominate private debts.

5 Conclusion

Our theory of the liquidity force that generates and sustains currency dominance rationalizes the historical emergence and persistence of regimes over the last four centuries as well as the transitions between them. The key feature that distinguishes the Dutch florin, the British pound, and the US dollar during their respective periods of dominance is the depth of the financial markets in those currencies. Whether this depth was intentionally created through investing in financial technologies, as in the Dutch case, or it was the outcome of global disruptions in geopolitical power as in the British and American cases, a large homogeneous pool of safe government-backed securities has been the springboard for each dominant currency. Once dominance is established, the ability and incentives to invest in institutions that facilitate financial market depth are greater in the center country and lead to greater entrenchment of the dominant regime.

In the second half of the twentieth century, the US dollar was established as the central currency of the international monetary system, and today, the dollar remains the de facto center. Looking into the future, our model has two implications for a modern Triffin dilemma in which world growth may be outpaced by the ability of the US government to create safe dollar liabilities. First, as in Despres, Kindleberger and Salant (1966), financial institutions that speed up settlement, such as well-functioning repo markets, reduce the burden on government debt as the primary source of liquidity. Second, private firms, both domestic and foreign, contribute to the stock of dollar-denominated money-like liabilities. However, we recognize that the latter does so by accumulating currency mismatch risk on balance sheets around the world, which can have costly financial spillovers, as discussed in Jiang et
Therefore while US GDP may not be the limiting factor, the world continues to face barriers in costlessly creating safe liabilities in a foreign denomination.

An often-asked question in academic and policy discussions is whether the Euro or the Chinese Renminbi (RMB) might be poised to displace the US dollar as the world’s dominant international currency in the near future (Horn et al. 2021, Clayton et al. 2022). There is evidence that in the early 2000s the Euro was challenging the dollar as a reserve currency, but this pattern reversed after the sovereign debt crises in Europe (Maggiori et al. 2019). Our paper ties this rise and fall to the quantity of safe assets in the Euro system, which grows upon the formation of the Euro, but then falls as some of the sovereign issuers in the Euro area are deemed unsafe after the debt crises. Likewise, our theory suggests that China’s safe debt markets are currently not sufficiently liquid, safe, or investable in size to challenge the dollar as a reserve currency.

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31 On the topic of over-borrowing externalities in an open economy context, see also Caballero and Krishnamurthy (2001), Lorenzoni (2008), Bianchi (2011), and Korinek (2018).
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A Additional Microfoundations and Proofs

A.1 Microfoundation forSwitching Cost $K_i$

Assume that at date $t_1$, the foreign exchange rate $E$ either appreciates or depreciates with equal probability. Thus, a firm in country $B$ can convert its revenue to currency $A$ at a stochastic FX rate $E \in \{1 + \gamma, 1 - \gamma\}$. Suppose that a firm in $B$ chooses to issue debt in $A$. At $t_1$, the exchange rate realizes, with a depreciation of currency $B$ to $1 - \gamma$ being a bad state for the firm: the firm has revenues in units of currency $A$ of $1 - \gamma$ and debt obligation of one. We assume that in this bad state, the firm-i can pay a disutility cost of $\kappa_i \gamma > 0$ to make up for the lost revenue. The disutility cost is a modeling device that ensures that firms reckon some cost due to currency mismatch, and to ensure that the firm does not default so that the bond is riskless (and hence private bonds are perfect substitutes for government-issued bonds). We assume that there is heterogeneity in this cost across firms. In this case,

$$K_i = \frac{1}{2} \gamma \kappa_i \quad (A.1)$$

which readily maps back to the model in the main text.

A.2 Proofs Not Included in Main Text

This section contains proofs that are omitted from the main text.

**Proof of Proposition 6.** The equilibrium masses of firms are:

$$m_{F,A} = \rho_A \phi \left[F_A + H(\hat{K})F_B\right], \quad m_{F,B} = \rho_B \phi \left(1 - H(\hat{K})\right)F_B, \quad (A.2)$$

while the masses of liquidity suppliers are,

$$m_{I,A} = G_A + \rho_A F_A + \rho_A H(\hat{K})F_B, \quad m_{I,B} = G_B + \rho_B \left(1 - H(\hat{K})\right)F_B. \quad (A.3)$$

Consider the objectives for country $A$ and $B$. We have that

$$W_A = G_A(\lambda_A m_{F,A}) + \rho_A F_A \lambda_A (m_{F,A} + \phi m_{I,A})$$

$$= G_A \lambda_A \rho_A \phi \left[F_A + H(\hat{K})F_B\right] + \rho_A F_A \lambda_A \left[\rho_A \phi \left[F_A + H(\hat{K})F_B\right] + \phi(G_A + \rho_A F_A + \rho_A H(\hat{K})F_B)\right]$$

A.1
and,

\[
W_B = G_B \lambda_B m_{F,B} + \rho_B F_B (1 - H(\hat{K})) \lambda_B (m_{F,B} + \phi m_{I,B}) + U_{B \to A}
\]

\[
= G_B \lambda_B \rho_B \phi \left( 1 - H(\hat{K}) \right) F_B \\
+ \rho_B F_B (1 - H(\hat{K})) \lambda_B \left( \rho_B \phi \left( 1 - H(\hat{K}) \right) F_B + \phi (G_B + \rho_B \left( 1 - H(\hat{K}) \right) F_B) \right) + U_{B \to A}
\]

It is straightforward to see that,

- \( \frac{\partial W_A}{\partial \rho A} > \frac{\partial W_B}{\partial \rho B} \) since \( G_A > G_B, \lambda_A \geq \lambda_B \) and \( F_A \geq F_B \).

- \( \frac{\partial^2 W_A}{\partial \rho A \partial \hat{K}} > 0 \) since \( \frac{\partial m_{F,A}}{\partial \hat{K}} \) and \( \frac{\partial m_{I,A}}{\partial \hat{K}} \) are positive and \( \frac{\partial \hat{K}}{\partial \rho A} \) is positive.

\[ \square \]

**Proof of Proposition 7.** Global welfare is \( W = W_A + W_B \). Call \( K^* \) the global welfare optimizing \( K \). We wish to show that \( K^* > \hat{K}_{\text{max}} \). It is sufficient to show that,

\[
\left. \frac{\partial W}{\partial K} \right|_{K=\hat{K}} > 0.
\]

(A.4)

We have,

\[
\frac{\partial W_A}{\partial K} = G_A \lambda_A (\phi F_B h) + F_A \lambda_A 2(\phi F_B h),
\]

(A.5)

and,

\[
\frac{\partial W_B}{\partial K} = -G_B \lambda_B (\phi F_B h) - (1 - H) F_B \lambda_B 2(\phi F_B h) \\
- \lambda_B (m_{F,B} + \phi m_{I,B}) h F_B + \lambda_A (m_{F,A} + \phi m_{I,A}) h F_B - \hat{K} h F_B \\
+ F_B H \lambda_A 2(\phi F_B h).
\]

Notice that the middle line here is equal to zero: it is the equilibrium condition that determines \( \hat{K} \). Then,

\[
\left. \frac{\partial W}{\partial K} \right|_{K=\hat{K}} = \{ G_A \lambda_A - G_B \lambda_B + 2F_A \lambda_A - 2F_B \lambda_B + 2F_B H \lambda_A + 2F_B H \lambda_B \} (\phi F_B h),
\]

(A.6)

which is positive in the case of \( A \) dominance \( (G_A > G_B, \lambda_A \geq \lambda_B) \).

\[ \square \]

**Proof of Proposition 9.** The relevant derivations are given in Appendix Section A.4.

\[ \square \]

**Proof of Proposition 3.** Differentiating equation (33), we obtain:

\[
\frac{\partial \hat{K}^+(\hat{K}^-)}{\partial \hat{K}^-} = \phi (\lambda_A + \lambda_B) h (\hat{K}^-) F_B^{-},
\]

(A.7)

\[ ^1 \text{We also need } W \text{ to be globally concave for this approach to be valid. It is straightforward to differentiate } \frac{\partial W}{\partial K} \text{ and show that the second derivative is negative as long as } h' < 0, \text{ which the Pareto distribution satisfies.} \]
where \( h(\cdot) = H'(\cdot) \) is the probability density function (PDF) for the distribution of costs \( K_i \). We assume that the density has non-zero mass over the entire the support \([K, \infty)\), which is the case for the Pareto distribution that we use to derive our analytical results. Since \( h \) is a PDF, it is then positive over the full support: \( h(\hat{K}^-) > 0 \). All other terms are also positive (\( \phi, \lambda_A, \lambda_B, F_F^- > 0 \)), so

\[
\frac{\partial \hat{K}^-(\hat{K}^-)}{\partial \hat{K}^-} > 0.
\]  
(A.8)

By the same argument, differentiating (34) we have

\[
\frac{\partial \hat{K}^-(\hat{K}^-)}{\partial \hat{K}^-} = \phi(\lambda_A + \lambda_B)h(\hat{K}^-)F_F^+ > 0.
\]  
(A.9)

\section*{A.3 Conditions for Equilibrium Existence and Convexity}

This section discusses the formal conditions required for equilibrium existence as well as the second-order condition associated with the optimization problems that feature in the model. We derive conditions on the cost distribution \( H(K_i) \) under which the model’s objective function is well-behaved, and we show that the Pareto distribution featured in our baseline analysis satisfies these conditions.

Consider equilibria of class \( BA \) (as the analysis is symmetric for \( AB \) equilibria). The marginal firm with \( K_i = \hat{K} \) satisfies

\[
\lambda_A (m_{F,A} m_{I,A})^{\theta-1} [m_{F,A} + \phi m_{I,A}] - \hat{K} = \lambda_B (m_{F,B} m_{I,B})^{\theta-1} [m_{F,B} + \phi m_{I,B}],
\]  
(A.10)

while the market clearing conditions are

\[
m_{F,A} = \phi \left[ F_A + H(\hat{K})F_B \right], \quad m_{F,B} = \phi \left( 1 - H(\hat{K}) \right) F_B,
\]  
(A.11)

and

\[
m_{I,A} = G_A + F_A + H(\hat{K})F_B, \quad m_{I,B} = G_B + \left( 1 - H(\hat{K}) \right) F_B.
\]  
(A.12)

Define

\[
\Delta(\hat{K}) = \lambda_A (m_{F,A} m_{I,A})^{\theta-1} [m_{F,A} + \phi m_{I,A}] - \lambda_B (m_{F,B} m_{I,B})^{\theta-1} [m_{F,B} + \phi m_{I,B}].
\]  
(A.13)

The equilibrium condition is that

\[
\Delta(\hat{K}) = \hat{K}.
\]  
(A.14)

Consider the \( \theta = 1 \) case. Then

\[
\Delta(\hat{K}) = \lambda_A [m_{F,A} + \phi m_{I,A}] - \lambda_B [m_{F,B} + \phi m_{I,B}].
\]  
(A.15)
Substituting in the masses:

\[
\Delta(\hat{K}) = \phi \lambda_A \left[ F_A + H(\hat{K}) F_B + G_A + F_A + H(\hat{K}) F_B \right] - \phi \lambda_B \left[ 2 \left( 1 - H(\hat{K}) \right) F_B + G_B \right],
\]

(A.16)
or

\[
\Delta(\hat{K}) = \phi \lambda_A \left[ 2 F_A + G_A \right] - \phi \lambda_B \left[ 2 F_B + G_B \right] + 2 \phi F_B (\lambda_A + \lambda_B) H(\hat{K}).
\]

(A.17)

Under A dominance \((F_A > F_B, G_A > G_B, \lambda_A > \lambda_B)\), the term marked \((1)\) is positive and constant. The term marked \((2)\) is zero at \(\hat{K} = \hat{K}\), and it asymptotes to \(2\phi F_B (\lambda_A + \lambda_B) > 0\) as \(\hat{K}\) goes to infinity. The second term is also strictly increasing.

To get two crossings then we only need that \(H(\cdot)\) is sufficiently curved for low values of \(\hat{K}\). A necessary condition is that

\[
\frac{d}{d\hat{K}} \left( 2\phi F_B (\lambda_A + \lambda_B) H(\hat{K}) \right) > 1,
\]

(A.18)
or

\[
2\phi F_B (\lambda_A + \lambda_B) h(\hat{K}) > 1
\]

(A.19)

for low values of \(\hat{K}\).

Candidate distributions are ones for which

\[
h(\hat{K}) > \frac{1}{2\phi F_B (\lambda_A + \lambda_B)},
\]

(A.20)

and where \(h(\cdot)\) is uniformly decreasing. The Pareto distribution satisfies these criteria, as do many other distributions.

### A.4 Derivations for the Model With Aggregate Risk

This section provides additional derivations for the extended version of the model with aggregate risk that we introduce in Section 4.5. We analyze this extended version of the model in the \(\theta = 1\) case. To start, we consider the equilibrium at time \(t_1\) in state \(\omega\). The masses of liquidity providers are

\[
m^{\omega}_{I,j} = G^{\omega}_j + F_j
\]

(A.21)

where we recall that we allow for state-contingency in the supply of government assets \(G^{\omega}_A\), but we assume that \(F_j\) is not state-contingent. Correspondingly, the masses of liquidity demanders are

\[
m^{\omega}_{F,j} = \phi^{\omega} F_j.
\]

(A.22)

The two-sided match probabilities are then

\[
\alpha^{\omega}_{F,j} = \lambda_j m^{\omega}_{I,j}, \quad \alpha^{\omega}_{I,j} = \lambda_j m^{\omega}_{F,j}
\]

(A.23)

\(P(\text{Buyer finds a seller}) \quad P(\text{Seller finds a buyer})\).
The surplus from are match remains $1 - \beta$, independent of the aggregate state. The date $t_0$ price of the private bond is therefore

$$
P_{0,j} = E \left[ \alpha_{I,j} \beta \left( 1 - \eta \right) \left( 1 - \beta \right) \right] + \frac{(1 - \alpha_{I,j}) \beta^2}{P(\text{Not Matched}) \times \text{PV of 1}}.
$$

which we rewrite as

$$
P_{0,j} = \beta^2 + (1 - \eta) \lambda_j E[m_{F,j}^\omega] \beta(1 - \beta).
$$

Firm utility at date $t_0$ is

$$
u_{i,j}^F = P_{0,j} + \beta E[\phi^\omega m_{I,j}^\omega] \eta(1 - \beta),
$$

which, substituting in for $P_{0,j}$, becomes

$$
u_{i,j}^F = \beta^2 + (1 - \eta) \lambda_j E[m_{F,j}^\omega] \beta(1 - \beta) + \beta \lambda_j E[\phi^\omega m_{I,j}^\omega] \eta(1 - \beta).
$$

As before, we take the case $\eta = \frac{1}{2}$ and rewrite:

$$
u_{i,j}^F - \beta^2 = \frac{1}{2} \lambda_j \beta(1 - \beta) \left( E[m_{F,j}^\omega] + \phi^\omega m_{I,j}^\omega \right).
$$

This expression is similar to that of the non-stochastic case, except that the masses are now stochastic.

We next substitute in for the masses and rewrite:

$$
u_{i,j}^F - \beta^2 = \frac{1}{2} \lambda_j \beta(1 - \beta) \left( E[\phi^\omega (2F_j + G_j^\omega)] \right)
$$

This yields the indifference condition presented in the main text:

$$
\lambda_A \left( E[\phi_\omega] \left( 2(F_A + H(\hat{K})F_B) + E[G_A^\omega] \right) + \text{Cov}[\phi_\omega, G_A^\omega] \right) - \hat{K} = \lambda_B E[\phi_\omega] \left( 2(1 - H(\hat{K}))F_B + G_B \right).
$$

B Further Historical Details

This section provides additional details which supplement our discussion of the various historical episodes.

B.1 Bank of Amsterdam

The Dutch florin created by the Bank of Amsterdam in 1609 was the first global currency. For much of history, transactions and debts around the world were primarily settled in metallic coins.
However, hundreds of domestic and foreign varieties existed, and using them entailed large transaction costs such as transportation, insurance, and assayance. The difficulty of enforcing quality created incentives to debase the currency and reduce the circulating supply of high-quality coins. These costs compounded the difficulty of coordinating on the coins that were valid for settling a debt. While negotiable credit instruments such as the bill of exchange reduced the need to transfer coins, they still required a unit of denomination and so settlement ultimately relied on an uncertain supply of physical assets.

The Bank of Amsterdam was chartered by the City of Amsterdam to provide a high quality standardized currency that would reduce settlement frictions. The Bank primarily did so by creating a currency that existed on its ledgers (“bank florin”) that was backed by coin and could be transferred across accounts freely. The City of Amsterdam initialized the pool of florin available for settlement by requiring that all large bills of exchange drawn and/or payable in Amsterdam had to be settled at the Bank, i.e., denominated in florin. Relative to the uncertain supply of specific metal coins in circulation anywhere else in the world, the Bank provided florin the advantage that there would be a ready supply for payments in Amsterdam.

Bank accounts were freely provided to anyone, and florins were credited to accounts for deposits of recognized coins. These coins backed the florins that could be withdrawn as current guilders in a narrow bank model. Since the Bank charged a fee for withdrawals, it was usually less costly to trade florin for current guilder in a secondary “open market.” In that market, the agio was the market exchange rate between the bank florin and the current guilder.

Rotterdam, a neighboring mercantile city, also created its own exchange bank modeled after the Bank of Amsterdam. While the two institutions maintained separate balance sheets, Rotterdam adopted Amsterdam’s agio because merchants preferred florin (Van der Borght, 1896, p. 209). Rotterdam provided a system where all deposits and withdrawals of guilders were made allowing for the Amsterdam agio and thus used Amsterdam’s florin as the unit of account (Carey, 1818, p. 369). Rotterdam also conceded to provide current accounts, which were the primary means for merchants to access florin by way of guilders and much more heavily used than its own bank money (Van der Borght, 1896, p. 210). In addition, the Bank of Rotterdam, despite requiring large bills of exchange to be settled in its own bank florin, also settled bills payable in florin in Amsterdam. In these ways, Rotterdam provided access to florins to the extent possible given its separate balance sheet.

In 1683, the Bank also introduced a receipts technology that operated like a modern day repurchase agreement. The Bank of Amsterdam advanced florin for short-term deposits of specie

---

2An ordinance in the Dutch Republic from 1606 officially recognized 25 gold and 14 silver trade coins from 35 domestic mints, but many more varieties circulated, and the Republic officially published exchange rates for almost 1000 coins (Roberds and Velde, 2016, p. 344).

3The first ordinance in February 1609 applied to bills over 600 guilders; in 1642 year this was revised to include bills over 300 guilders. As a result, all merchants kept an account at the Bank, and the Bank maintained two to three thousand accounts at any given time (Van Dillen, 1934, p. 107).

4The withdrawal fee of 1.5% covered the costs that the Bank of Amsterdam incurred to mint current guilders for deposits of inferior coins (judged by their metallic content).
and metal bars. Depositors were issued a receipt, negotiable and renewable with an initial maturity of six months, for the right to withdraw the specific metal they deposited.\(^5\) This technology broadened the set of assets that could be converted into florin beyond the original set of trade coins, and it was beneficial for both the Bank of Amsterdam and for private parties. The former gained from the metal deposits, which became part of the Bank’s assets if the receipt expired, and the latter was able to obtain florin for settlement without needing to convert them into eligible trade coins for deposit at market value.\(^6\)

Following the introduction of the repo facility, the quantity of florins at the Bank of Amsterdam doubled from approximately eight million to sixteen million from the mid 17th to the beginning of the 18th century (Quinn and Roberds, 2014b). These complementary innovation forces are also at play in the rise of Amsterdam. For instance, the receipts technology (i.e., repurchase facility) that created florin claims out of raw specie was only introduced in 1683, almost seven decades after the Bank’s establishment. The fact that florin balances doubled after its introduction is indicative of the force identified in our model that increases in \(\lambda_j\) generate entry (Quinn and Roberds, 2014b).

After a century of dominance, the Bank of Amsterdam eventually became a victim of its own success. Intermediation in bank florins was profitable for the Bank, and it routinely turned over its wealth to the City of Amsterdam, leaving it with little capital buffer. It also made advances to the Dutch East India Company (VOC), which eventually led to runs on bank florin after the VOC came close to failure following the fourth Anglo-Dutch War in 1784. The French invasion in 1795 led to a drop in the \(\text{agio}\) to -14\%, after which it never fully recovered, and eventually the Bank was formally dissolved in 1819.

### B.1.1 Dutch versus Spanish Trade

The figures on Spanish trade per capita and the population are taken from Ortiz-Ospina et al. (2018) and Allen (2003) on p. 438 (Table A1) respectively. Data are available for four years (1600, 1700, 1750 and 1800). Dutch trade comes from Zanden and Leeuwen (2018) survey. We extract the four data points to compute Spanish trade relative to Dutch trade in the 17\(^{th}\) – 19\(^{th}\) centuries. Pound-guilder exchange rates are taken from Denzel (2017) (Figure 28.1).

In order to compare Spanish trade to Dutch trade volumes, we need the estimates for Spanish trade in guilder. Thus, we multiply the Spanish trade in £ per capita by Spanish population and divide it by the £–guilder exchange rate to arrive at the final figures below.

---

\(^5\)There was a large secondary market in receipts. Receipts could be redenominated in smaller face values, and they were renewable by paying the withdrawal fees. Withdrawal fees with receipts (0.125\% for silver and 0.25\% for gold) were much lower than that for current guilders (1.5\%) because the Bank did not need to mint guilders to meet withdrawal demands. Around this time it appears the Bank of Amsterdam eliminated the right to withdraw from its accounts, which has led some authors to argue that the florin was an early fiat currency (Quinn and Roberds, 2014b).

\(^6\)Given the wide variety of specie circulating, the demand and supply for specific coins varied significantly, and market prices were usually in flux. The receipts technology made it possible to transact on the Bank’s mandated value for the specie while retaining the ability to withdraw and sell at a future date when prices rose. It also supported a large trade in precious metals since the freely-traded receipts were equivalent to advances on pledges of the underlying metals.
Table B.I: Estimates to compute Spanish trade

<table>
<thead>
<tr>
<th>Year</th>
<th>Holland (in million guilder)</th>
<th>Trade Spain (in £ per capita)</th>
<th>Population Spain</th>
<th>Exchange rate £-guilder</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
<td>13.462</td>
<td>.18907407</td>
<td>8,700,000</td>
<td>0.095</td>
</tr>
<tr>
<td>1700</td>
<td>12.152</td>
<td>.31009123</td>
<td>8,600,000</td>
<td>0.1</td>
</tr>
<tr>
<td>1750</td>
<td>10.221</td>
<td>.57871836</td>
<td>9,600,000</td>
<td>0.095</td>
</tr>
<tr>
<td>1800</td>
<td>16.241</td>
<td>.48912659</td>
<td>13,000,000</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table B.II: Trade volumes (in million guilder)

<table>
<thead>
<tr>
<th>Year</th>
<th>Holland</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
<td>13.462</td>
<td>17.315</td>
</tr>
<tr>
<td>1700</td>
<td>12.152</td>
<td>26.667</td>
</tr>
<tr>
<td>1750</td>
<td>10.221</td>
<td>58.481</td>
</tr>
<tr>
<td>1800</td>
<td>16.241</td>
<td>70.651</td>
</tr>
</tbody>
</table>

B.2 Great Britain

This initial point established the pound as a dominant currency with substantial entry from foreign sovereigns. The institutional developments in the legal structure of bills of exchange and the role of the Bank of England also reflect investments in overall market liquidity $\lambda_A$. These investments continued throughout the 19th century as dominance engendered entry, liquidity, and increased incentives to innovate.

The Bank of England was a key institution in the London money market, founded in 1694 as a note-issuing private corporation that was granted several privileges in return for raising and administering the Crown’s debt. During the early part of its history, the Bank competed with other private banks to increase its note circulation and raise its profits. In this respect, the Bank was like any other private firm that was incentivized to issue safe debt in order to benefit from the yield premium in equation (A.25). It was very successful in establishing a sound reputation for its notes, and by the late 18th century, Bank notes became synonymous with the pound sterling (Thornton, 2017).

---

7The vast majority of the sovereign debt issued in London after the Napoleonic Wars were by Latin American and other European nations rather than the British colonies (Meyer et al., 2022).

8Like most currencies during this era, the pound sterling referred to specific metallic coins, and obligations denominated in sterling were contracted to be repaid in those coins. However, coins were inconvenient for the reasons already discussed, and private banks found it profitable to issue paper notes denominated in sterling (i.e., claims on sterling coin). The privileges restricted banking competition and gave the Bank of England a monopoly over note issuance. From 1697 until 1844, only the Bank of England could raise equity; all other banks were restricted to partnerships of six or fewer (after 1844, this was altered to a radius of 25 miles around London). In 1708, the Bank was granted an exemption to laws restricting bank note issuances to private partnerships (Broz and Grossman, 2004).

9The Banking Act of 1826 required the Treasury to monitor the amount of small-denomination notes issued by the Bank following the 1825 crisis in which the Bank was seen to have abused its monopoly (Scammell, 1968, p. 132).

10Estimates of historical convenience yields in the era of British pound dominance include Chen et al.
In 1833 and subsequently 1844, the de facto equivalence between the pound and Bank notes became de jure with passage of the Bank Notes Act and the Bank Charter Act respectively. The former made Bank notes legal tender while the latter consolidated the entire note issuance onto the Bank of England’s balance sheet where it was fully backed in gold reserves above the allowed fiduciary issue. The full note circulation of the Bank of England therefore officially contributed to the supply of $G_A$ following the 1840s.

A second important innovation was the legal codification of the contractual terms for bills of exchange, which coordinated the market on the terms of borrowing and the procedures for default. Bills of exchange were the primary London money market instrument, and each time one was traded (“discounted”), the seller guaranteed (“endorsed”) the bill. These endorsements were legally equivalent to being the original borrower, and so each endorser was equally liable. The generality of these conditions were constantly tried at court and established a strong legal precedent. These laws reduced the information sensitivity of bills and collectively raised the safety and liquidity of all bills of exchange with multiple endorsers, regardless of the idiosyncratic characteristics of the ultimate borrower. Thus, these innovations made private debt money-like in the sense of Dang et al. (2017).

A third notable institution was the Bank of England’s role as a credible and reliable lender of last resort to the financial sector. During the banking crises of 1847, 1857, and 1866, the Bank obtained permission from the Treasury to suspend the Bank Charter Act in order to meet all demand for Bank notes. In fact, the Bank’s behavior during the crisis of 1866 was the basis for Bagehot’s rules for central banking (Bagehot, 1873). As a lender of last resort, the Bank provided liquidity at its discount window by converting private bills of exchange into pounds sterling, thereby de facto became a backstop to the private bills market. This backstop officially only applied to high-quality bills—those first guaranteed (“accepted”) by large merchant banks that held accounts at the Bank of England—but like all liquidity backstops, its existence reduced the occurrence of market freezes and increased the willingness of private firms to lend in all states. These forces

---

11 The limits of the Bank’s note supply was therefore primarily governed by the gold reserves at the bank and secondarily by the government-determined fiduciary issue. Private bank notes already in circulation were allowed to remain, but no new notes could be issued, and banks lost their right when they merged. The Bank Charter Act could be suspended during financial crises when there was large demand for Bank notes. A Parliamentary report from 1837 describes the legal protections against default: “a holder of a bill of exchange can bring actions at one and the same time, against every party whose name is attached to it, and in the event of the failure of them all, can prove upon the estate of each for the full value of the bill” (Joplin, 1837, p. 17).

12 An additional factor is that the Act of 1833 exempted short-term bills of exchange from the Usury Laws, which also expanded the market’s general willingness to hold them (Scammell, 1968).

13 Since the Bank Charter Act limited the supply of Bank notes to the Bank’s gold reserve, suspending it (and therefore the gold standard) was the only way to ensure they could meet demand. Even when the gold reserve was high, the presence of a limit reduced liquidity in the market. It is worth noting that obtaining permission to suspend the Bank Charter Act was sufficient, and Great Britain did not actually suspend the gold standard during this period.

14 The high quality bills eligible at the Bank of England became a class of their own, and the financial press throughout this period reported the rates on “Bank” bills separately from “trade” bills (Xu, 2022).
together led issuers to prioritize denomiating issues in sterling, thereby increasing the quantity of safe pound-denominated debt in the London money market.

The Bank of England acting as a lender of last resort was a major transition from its earlier history in the 18th century when discounting and note issuance was a profit-maximizing endeavor. At that point, the Bank’s discount window followed the market and became similarly unavailable during downturns and crises.\textsuperscript{16} As the London money market deepened and the pound sterling gained dominance in the 19th century, the Bank increasingly took on a more formal role within the government. This was despite the fact that it remained privately owned by stockholders until 1944 and run by Governors and Courts of Directors that primarily stemmed from the merchant and banking classes (Cassis, 1994, p. 85). The Bank’s transitioning role in the money market given its dual identities reflects how the benefits of agglomeration accrued to both the government and to the private sector, all embodied in a single institution.

One final development during this period that contributed to maintaining the dominant equilibrium is the growth of international banking, which facilitated access to the pound sterling in locations around the world. British overseas banking institutions generally followed the business model of issuing deposits and shares domestically while lending via bills of exchange payable in London in their branches abroad. As in Amsterdam, the short term commercial bill became the dominant credit instrument internationally, with payments settled in London even for transactions that did not involve Great Britain.\textsuperscript{17} The network of British banks increased the likelihood that foreign firms could hold pound obligations (or equivalently receive part of their profits in pounds), which in the context of the model we view as equivalent to reducing the cost of foreign currency issuance $K_i$, whether through a reduction of underlying FX exposures or via a reduction in the fixed cost of debt underwriting. Reducing this cost increases the mass of firms for which issuing in the foreign currency is profitable, as shown in equation (A.10).\textsuperscript{18}

\textsuperscript{16}Scammell writes, “All in all the discounting of bills by the Bank in the early 19th century must be seen primarily as a prosperous business of the Bank and only very secondarily as a manifestation of credit policy,” (Scammell, 1968, p. 144). For example, during the 1797 crisis, Parliament assumed the role of being a liquidity provider by issuing exchequer (treasury) bills to the market (Thornton, 2017, p. 98). The subsequent crisis in 1825 provides a microcosm into the transition that took place. Early in the year, the Bank of England closed its discount window because it anticipated a financial market downturn. This action in itself ”created an atmosphere of misgiving and potential crisis,” (Scammell, 1968, p. 131). However, when the crisis peaked in November with numerous failures in London, the Bank reversed its earlier decision and made discounts and advances on government securities and private bills. Thereafter starting in 1830, it allowed bill brokers to access the discount window for the first time, after recognizing that these institutions were important conduits of liquidity.

\textsuperscript{17}For example, “the bill on London enabled the banks [...] to finance a large share of international trade regardless of whether that trade touched Britain’s shores,” (Orbell, 2017, p. 8), and “wines from France, coffee from Brazil, sugar from the West Indies, and silk from Hong Kong were paid alike with bills on London,” (Jenks, 1927, p. 69).

\textsuperscript{18}Incidentally, both the French and the Germans followed the British model, often with explicit reference to expanding their currencies abroad. For instance, Edward Hurley, in his arguments for the creation for the US Federal Reserve System wrote, “The logical ambition of the German commercial policy is naturally to enthrone the Mark in the estimation of the world until it need pay no deference to the pound sterling.”

A.10